Teaching Students with Mild Intellectual Disability to Solve Word Problems Using Schema-Based Instruction

Irene Polo-Blanco*, María J. González*, Alicia Bruno**, Jon González-Sánchez***

*Universidad de Cantabria

**Universidad de La Laguna

***Universidad del País Vasco

Author Note

Irene Polo-Blanco: irene.polo@unican.es

María J. González: mariaj.gonzalez@unican.es

Alicia Bruno: abruno@ull.edu.es

Jon González-Sánchez: jon.gonzalez@ehu.es

Corresponding author: Irene Polo-Blanco, Departamento Matemáticas Estadística y Computación, Facultad de Ciencias, Universidad de Cantabria, Avda de los Castros s/n, 39005, Santander (Spain)

Acknowledgments. This work was supported by the Ministerio de Economía y Competitividad. Madrid. Spain. Projects EDU2017-84276-R, PID2019-105677RB-I00

Teaching Students with Mild Intellectual Disability to Solve Word Problems Using Schema-Based Instruction

Abstract

This study, designed using a multiple baseline across students, examines the effectiveness of a modified schema-based instructional approach to improve the mathematical word-problem-solving performance of three students with mild intellectual disability, two of them with autism spectrum disorder. Following the intervention, the three students improved their performance when solving addition and subtraction change word problems, in particular, their performance in inconsistent change word problems. The effects of the instruction were generalized to two-step addition and subtraction word problems for the three participants. Moreover, the results were generalized to an untrained setting, and were maintained eight weeks after the instruction. The implications of these findings for teaching problem-solving skills to students with intellectual disability, in particular to those with autism spectrum disorder, are discussed.

Teaching Students with Mild Intellectual Disability to Solve Word Problems Using Schema-Based Instruction

In different countries, many students with intellectual disability learn mathematics in general and/or special education classrooms. It is important to know methods for teaching this subject that may help them to progress not only in terms of academics, but also in terms of using mathematics outside the classroom, so that they can gain independence in their daily lives (Kasap & Ergenekon, 2017). More information is needed on how they learn mathematics and how to provide effective instruction that is adapted to their needs (Bowman et al., 2019; Desmarais et al., 2019; Xin, 2019).

In previous decades it was thought that students with intellectual disability only learn mathematical procedures and, to a lesser extent, gain an understanding of the concepts, which is why the emphasis in their teaching-learning process has often focused on practicing mechanical exercises (Baroody, 1999; Jitendra et al., 2002). This implies that they cannot advance in the matter itself, as this requires a conceptual understanding. However, this belief seems to be changing in recent years, at both the classroom and research level (Bowman et al., 2019).

Students with Mild Intellectual Disability (MID), meaning those with an IQ between 50-75 according to the *American Association on Intellectual and Developmental Disabilities* (2011), are also likely to experience slow academic growth, exhibit low performance in many academic areas, have difficulties with problem solving and abstract reasoning, and experience deficits in working memory (Hord & Bouck, 2012). Some studies have shown that students with MID can achieve success in mathematics if they receive appropriate instruction (Bowman et al., 2019), although there is still a lack of knowledge and attention to evidence-based mathematical practices (Bouck et al., 2018).

Arithmetic word problems constitute one of the main tasks in the teaching of mathematics. Solving them requires creating a mental representation of the word problem structure, selecting the appropriate arithmetic operation to find the unknown, executing the selected operation, reactivating the mental representation by inserting the answer that was calculated, and verifying if the answer is correct (De Corte et al., 1985).

Students with intellectual disability (ID) and/or autism spectrum disorder (ASD) have difficulties following these steps and need adaptations that help them understand the information given (for example, using a simple linguistic structure or familiar vocabulary) and visual aids during the problem-solving process (Bruno et al., 2021; Jitendra et al., 2002; Polo-Blanco et al., 2022).

Researchers have shown that students with ID and/or ASD improve their problem solving when they follow an adapted instruction, such as the schema-based instruction methodology (SBI) (Alghamdi et al., 2020: Desmarais et al., 2019; Gevarter et. al., 2016; Jitendra et al., 2002; Kasap & Ergenekon, 2017; Rockwell et al., 2011; Xin & Jitendra, 1999). SBI is an instructional approach that uses schematic diagrams associated with problem structures to provide a visual representation that aids students and emphasizes conceptual understanding by creating links between the problem elements and its solution (Desmarais et al., 2019; Xin & Jitendra, 1999).

Addition and subtraction problems exhibit different structures called *combination*, *change* and *comparison* (Riley et al., 1983). Researchers have shown that addition and subtraction problems differ in their level of difficulty depending on their structure and the location of the unknown (Carpenter & Moser, 1984). This research focuses on change problems, which involve an initial quantity that is either increased or decreased by an action, resulting in a different final quantity. There are three types of change problems, depending on the location of the unknown quantity: FAU (final amount unknown), CAU (change amount unknown), and IAU (initial amount unknown). Each of these types can also be of type "get more" or "get less", depending on the action of the verb. For example, *John has 13 marbles. He lost 7 marbles. How many marbles does John have now?* is a "get less" problem of type FAU. For typically-developing students, it has been shown that the FAU problems are the easiest, followed by the CAU and the IAU problems (Carpenter & Moser, 1984; Riley et al., 1983). Problems can also be classified as *consistent* or *inconsistent.* Inconsistent problems are those in which the required arithmetic operation is contrary to the statement's relational term. For example, the following IAU problem: *Dan had some marbles. He found 9 more marbles. Now he has 15 marbles. How many marbles did he have to start with?* is a "get more" problem that requires a subtraction to find the solution, so it is an inconsistent problem (Lewis & Mayer, 1987). There are three types of inconsistent change problems, namely those of type: CAU "get more", IAU "get more" and IAU "get less".

Inconsistent problems have been extensively analyzed in the research literature that focus on linguistic factors that affect problem-solving performance for typically-developing students (Daroczy et al., 2015; De Koning et al., 2017; Hegarty et al., 1995; Lean et al., 1990; Lewis & Mayer, 1987; Pape, 2003). All these researchers have found that inconsistent word problems are more difficult than consistent problems for typically-developing students. This conclusion has been explained by the fact that many students base their solution plan on direct translation strategies in which they associate keywords with arithmetic operations – i.e., the keywords less or more with the arithmetic operation of subtraction or addition, respectively–, whereas successful strategies are based on constructing a model of the situation described in the problem.

In the case of students with learning difficulties or low performance, the consistency effect has been analyzed by a few researchers, who have also revealed that students experience more difficulties with inconsistent word problems. These studies have connected reading comprehension and word-problem solving development, and they have oriented instruction to improve students' language skills (De Koning et al., 2017; Fuchs et al., 2009; Schumacher & Fuchs, 2012; Van der Schoot et al., 2009). Recently, Fuchs et al. (2019) developed a model that connects reading comprehension and schema-based word-problem solving via oral language comprehension for students with comorbid difficulty in reading comprehension and mathematics. This approach is being tested in contexts using Spanish-speaking learners of English.

The consistency effect has been scarcely analyzed for students with ID. Parmar et al. (1996) compared arithmetic word problem performance in mild disability and general education students in grades 3 through 8, finding an overall significant difference for the disability and grade factors: students with disabilities performed at a considerably lower rate than did students without disabilities. Moreover, students with disabilities did not increase their scores across the grade levels at the same rate. Cawley et al. (2001) and Parmar et al. (1996) analyzed, in particular, IAU "get more" and IAU "get less" word problems, concluding that cue words misguide students with disabilities as, for example, they identify the word "left" with "subtract" in problems like the following: *A boy had 3 apples left after he gave 2 apples to a friend. How many apples did the boy start with?* (Parmar et al, 1996, p. 417).

In the case of students with ID and/or ASD, SBI has been shown effective for teaching addition and subtraction problems. For example, Rockwell et al. (2011) studied the use of SBI to teach the three types of addition and subtraction problems to a 10-year-old student with ASD. Kasap and Ergenekon (2017) analyzed its effectiveness when teaching comparison problems to three students with ASD ages 9, 11 and 14. Similarly, Desmarais et al. (2019) compared the effectiveness of using SBI to solve change problems with different unknown locations, in three groups of three students ages 7 and 8 with ID and ASD, who performed poorly in mathematics. The use of SBI improved problem solving in the three groups, and indicated the need to continue research in this direction, given the small sample sizes in the various studies. Jitendra and her collaborators have extensively tested the effectiveness of SBI in different levels and contexts. For example, concerning additive problems, Jitendra et al. (2013) showed that third-grade students at risk for mathematics difficulties (MD) outperformed students in a control group on a word problem solving test, although no significant effects were observed in the maintenance session eight weeks after the intervention.

Given the unique characteristics that are usually present in students with ASD, some researchers have incorporated evidence-based practices for students with this disorder to traditional SBI (Wong et al., 2015). Some authors resort to a modified schema-based instruction (MSBI) approach that incorporates the use of visual supports, task analysis (as a heuristic in place of a mnemonic) and systematic prompting to traditional SBI. For example, Root et al., (2016) demonstrated that MSBI was effective in teaching change problems to three students with ASD and moderate ID. Browder et al. (2018) used an MSBI approach that included pictorial task analysis, graphic organizers, and systematic prompting with feedback to teach addition and subtraction word problems to eight students with moderate ID. Along similar lines, Cox and Root (2020) demonstrated the efficiency of the MSBI approach to teach mathematical problem-solving flexibility and communication to two middle school students with ASD.

In an effort to supplement the existing evidence, in this paper we analyze how a MSBI approach helps improve the ability of students with MID to solve addition and subtraction change problems, focusing our attention on inconsistent problems. Specifically, we consider the following research questions: (1) Is the MSBI approach effective in teaching addition and subtraction change problems to students with MID? (2) In particular, is the MSBI approach effective when teaching inconsistent change problems? (3) Will improvement in problem-solving performance generalize to two-step problems? (4) Will improvement in problem-solving performance be maintained over time?

Method

Participants and Setting

Three male students with MID, two of them diagnosed with ASD, participated in the study. The inclusion criteria for the study were: (1) be diagnosed with MID, IQ between 50-55 and 70 (AAIDD, 2011), (2) exhibit difficulties solving addition and subtraction word problems of all types (change, combine and compare), as identified by their tutors, (3) obtain a minimum score of 50 in the *Test of Early Mathematics Ability* (*TEMA-3*, Ginsburg & Baroody, 2007) to ensure the pre-requisite knowledge of numerical addition and subtraction facts; and (4) obtain a score of at most 50% on an initial test with the three types of addition and subtraction change problems (varying the unknown), designed in this study. The *Test of Early Mathematics Ability* TEMA-3 is a validated test with a Cronbach's alpha of 0.92.

The three students were enrolled in the same special education center. Student A was a 14-year-old Caucasian male. He was diagnosed with autism in a social security unit by a child psychiatrist. The results of the *Childhood Autism Rating Scale* (Schopler et al., 1988) placed him in the range of mild to moderate autism. He had an IQ of 55 (WISC-IV). Since the age of 13, he had been in a combined education (special and general) program, attending the special education school where the study took place 3 days a week. At the start of the study, he obtained a score of 71 (mathematical age over 10 years) on the *TEMA-3* mathematical proficiency test (Ginsburg & Baroody, 2007), scoring 100% in number facts, number skills and number comparison, 88% in calculation skills and 71% in concepts. He scored 50% in the initial assessment with change problems.

Student B, a 13-year-old Caucasian male, was diagnosed with ASD by a child psychiatrist, and had an IQ of 54 (WISC-V). The results of the *Childhood Autism Rating Scale* (Schopler et al., 1988) placed him in the range of severe autism. He had been enrolled full time in a special education school since the age of 11. He obtained a score of 56 on the *TEMA-3* test (mathematical age of 7 years and 8 months), scoring 96% in number skills, 78% in number facts, 71% in concepts and 50% in number comparison and calculation skills. He scored 33% on the change problem test.

Student C, a 17-year-old Caucasian male, was identified as having an intellectual disability (IQ = 58, WISC-IV). He had been enrolled in a special education school full time since the age of 12. He scored 50 on the *TEMA-3* test (equivalent mathematical age of 7 years), with 96% in number skills, 60% in concepts, 50% in number comparison and calculation skills and 44% in number facts. He scored 10% on the change problems initial assessment.

Dependent and Independent Variables

The dependent variable reflects the students' success rate in solving addition and subtraction change word problems. Performance is measured through two indicators: explicitly identifying the arithmetic operation needed to solve the problem (identify if the numbers given in the problem have to be added or subtracted) and providing the correct numerical answer to the problem (correctly execute the arithmetic operation needed to obtain the numerical solution).

The independent variable is the problem-solving intervention delivered using a modified schema-based instruction (MSBI) approach (Browder et al., 2018; Cox & Root, 2020). This intervention is detailed in the procedure section.

Design and Data Collection

A multiple baseline design across students was employed to assess the effectiveness of the modified schema-based instruction on the performance of addition and subtraction change problems. The phases of the experiment included: baseline, instruction on each type of change problem sequentially introduced (first FAU, second CAU, third IAU), generalization (to an untrained setting and to two-step problems) and maintenance (eight weeks after instruction). In order to show evidence of the dependence between instruction and student performance, and in keeping with the multiple baseline design, the intervention was introduced to different participants at different points in time. Each student was introduced to the intervention after a stable baseline. A baseline was considered stable when there was no more than a one-point difference between the scores of the last two baseline probes.

Probes and Scoring

Two types of probes were conducted in the different phases of the experiment: one-step problem-solving probes and two-step problem-solving probes. One-step problem-solving probes were administered during the baseline, instruction, generalization to untrained settings and maintenance phases. These problem-solving probes consisted of six one-step change problems of the "get more" and "get less" varieties, which included the three change types (FAU, CAU, IAU). During the instruction phase, all the probes contained six change problems of the type being taught until that moment. Two-step problem solving probes consisted of change problems with a final, unknown amount. In order to include every possible combination of "get more" and "get less", these probes consisted of eight problems. Two of these probes were administered: the first one during the baseline phase, and the second one during the generalization phase.

All the problems were randomly ordered in each probe. The number sets considered in the problems contained addends with no result higher than 15, and they did not contain repeated addends (4+4=8) or sums equal to 10 (6+4=10) in order to avoid easy numerical combinations (Carpenter & Moser, 1984).

Considering the phases in the problem solving processes that involve in (1) understanding the vocabulary or the real situation to which the problem refers and select the appropriate arithmetic operation required for finding the unknown, and (2) execute and interpret the selected operation (De Corte et al., 1985), the scoring for the one-step problem probes was as follows:

each problem was worth a total of 2 points: 1 point for correctly identifying the arithmetic operation needed to solve the problem, and 1 point for providing the correct numerical answer to the problem. In the two-step problems, each problem was worth a maximum of 3 points: 1 point for correctly identifying each of the operations and 1 point for the correct result.

Procedures

Baseline. During baseline, the instructor provided the student with a sheet with the problems to be solved. He told the student to solve it as well as possible and encouraged him to ask questions about the meaning of the words in the problem he did not understand. No other feedback was provided.

Instructional procedures. Two teachers with more than eight years of experience teaching students with disability provided the instruction. Student A and student C were taught by the same instructor. Both instructors knew the students and provided them with extracurricular support. None of the instructors were the students' regular teachers. Before the experiment, the researchers explained the phases of the study to the instructors to ensure that the correct instruction process was followed. They were taught how to use the schemas using the self-instruction list, and how to structure the instructional sessions through a model, lead and test format (explained in detail below). One of the authors had weekly meetings with both instructors. During these meetings, the most important aspects of the previous session were discussed, and the instructors were provided with a script containing the teaching instructions for the following session. These scripts contained flexible instructions to adapt the instruction to the difficulties of the students (e.g.: "if the student does not identify the larger amount in the problem, use the manipulatives to model the situation"). The sessions were held early in the morning during school hours in a room without distractions. One session per week was held for each student.

The MSBI approach was adapted from Cox and Root (2020) and Browder et al. (2018) and included: (1) explicit instructions on using a task analysis (a self-instruction sheet that incorporated visual representations for each step), (2) use of schematic diagrams and manipulatives, and (3) use of systematic prompting and feedback. The self-instruction sheet that accompanied the stories and, later, the problems (see Figure 1), was designed following similar studies (Root et al., 2016) and incorporated pictograms[1] to provide visual support.

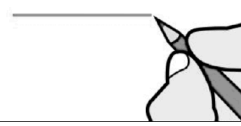
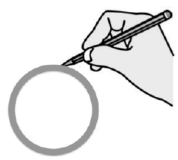
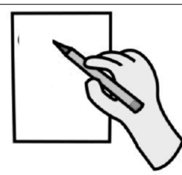
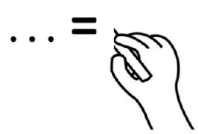
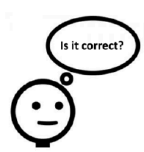
STEPS		TEACHER'S INSTRUCTIONS
1. READ AND UNDERLINE		"Read the problem and underline the verb. Is it a "get more" or a "get less" problem?"
2. CIRCLE THE LARGEST		"Circle the largest amount. If it is a get less problem, the initial amount is the largest one, if it is a get more problem, the final amount is the largest one."
3. FILL IN THE SCHEMA		"Fill in the schema with the amounts given in the problem, or leave it empty for the unknown."
4. FIND THE OPERATION	+ −	"Find the operation to solve the problem."
5. SOLVE		"Solve the operation to find the unknown of the problem and write the complete solution"
6. CHECK THE SOLUTION		"Check if the solution is correct. Read the problem again. Does it make sense?"

Figure 1. Self-instruction list for solving one-step change problems.

The instruction was carried out over two phases: phase I, the problem-schema instruction; and phase II, the problem-solving instruction (Xin, 2008). In phase I, change story situations were presented to students. No unknown information needed to be obtained; the students just had to place the numbers given in the problem into the correct place in the change schematic diagram. In phase II, change problems were presented to the students for them to solve.

As in similar MSBI studies (Cox & Root, 2020), the sessions followed a model, lead and test format. First, the instructor modeled how to follow the steps of the self-instruction sheet in order to fill in the schema for the given story or problem. After that, the instructor led the student in guided practice solving other change story situations (phase I) or problems (phase II). For each of them, the worksheet containing the story or the problem and the schematic diagram (see Figure 2) was provided to the student, together with the self-instruction sheet (see Figure 1).

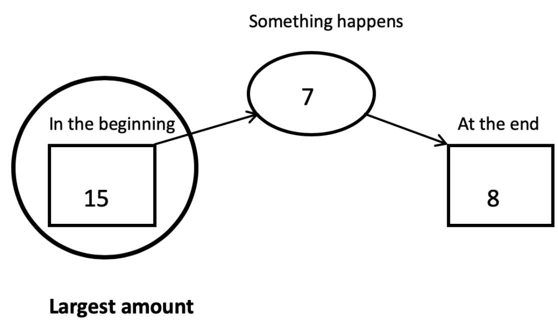


Figure 2. Change story diagram.

Finally, the instructor tested the student's ability to flexibly solve the stories (phase I) and problems (phase II) through independent practice. During this part, no feedback was provided to the students except for expressions of encouragement.

Phase I. Problem-schema instruction: change story situations. The students were instructed on the use of schematic diagrams for change story situations without an unknown (Figure 1). During this phase, the instructor showed the student how to follow steps 1, 2 and 3 of the self-instruction sheet (see Figure 1) in order to fill in the schema with the values provided in the story. In this way, the student first underlined the action (step 1). Next, the instructor asked the student to circle the largest amount (step 2). As in similar studies (Jitendra et al., 1999), the identification of the largest amount in the story was emphasized, noting the information provided by the action of the verb. Thus, in the story situation: *Pedro had 15 candies, he ate 7 and now he has 8*, since the verb implies "get less", they were asked: "when did he have more, at the beginning or at the end?" and then a circle was drawn around the larger quantity (Figure 2). Finally, the student placed the three numbers in the corresponding boxes of the schematic diagram (step 3). If the student showed difficulties understanding the story, the instructor used manipulatives to represent the situation. As in previous MSBI studies (Cox & Root, 2020), during this guiding phase, the instructor immediately provided error correction through modeling.

At the end of each session in change story situations, and after solving six stories guided by the instructor, the student independently solved another six stories, placing the numbers and circle in the corresponding places in the schematic diagram. Once the students mastered the use of schematic diagrams for stories (100% correct), phase II was commenced.

Phase II. Problem-solving instruction: change problems. Instruction on using schematic diagrams to solve each type of word problem in terms of the location of the unknown (FAU, CAU, IAU) was provided separately to each student, based on preliminary difficulty studies (Riley et al., 1983). As in phase I, the instructor first provided each student a demonstration and model for a change problem. In this case, the instructor followed the six steps of the self-instructional sheet in Figure 1 to show how to solve the problem. After that, the student was guided to solve a maximum of eight problems. These problems were solved by constantly interacting with the student, encouraging him to follow the six steps and explaining the meaning of the words he did not understand or correcting his mistakes. As in the previous phase, the situation in the problem was modeled using manipulatives when the student showed problems understanding it. At the end of each instructional session, the students solved a probe with six change problems, independently, without interacting with the instructor. If the student scored over 75% on the probe, the next problem type was introduced in the following instruction session. Details on the instruction on each problem type are provided below.

(1) Instruction on FAU problems: These sessions included instructional problems of type "get more" and "get less" with unknown the final amount. During the session, after reading each problem, the student placed the two numbers in the schematic diagram and wrote the question mark in the unknown position. Then, as was done in the instructional sessions on stories, he was asked to circle the larger amount (the teacher asked: "when is there more, at the beginning or at the end?") using the information given by the action of the verb. In this type of direct problem, which is more familiar to students, and in keeping with instructions from similar studies (Jitendra et al., 1999), emphasis was placed on the procedure to identify the operation in order to ingrain it for subsequent problem types. Thus, after placing the two numbers of the problem in the diagram and circling the larger amount, they were told: "if the question mark is in the circle, the two numbers have to be added; if there is a number in the circle, the two numbers have to be subtracted". The instructor interacted with the student during this process to verify the student's understanding of the steps.

After completing the instruction in each session, the student solved a probe with six FAU-type problems, three "get more" and three "get less".

(2) Instruction on CAU problems: The instruction sessions included FAU review problems and CAU ("get more" and "get less") problems. In these sessions, inconsistent problems (CAU "get more"), in which the operation is contrary to the action of the verb, appeared for the first time. To practice these problems, the instructor emphasized numbered steps 1 (Read and underline), 2 (Circle the largest), 4 (Find the operation) and 6 (Check the solution) (see Figure 1), in order to help them overcome the difficulties inherent to inconsistent problems. For example, in "Peter had 6 pencils, he bought some more and now he has 9. How many pencils

did he buy?" the student underlined the verb, and, since it was a "get more" verb, circled the final amount (9). The instructor indicated: the largest amount is circled. Since it is a number, the two numbers have to be subtracted to find the unknown: 9-6 = 3. Once the result was obtained, the student checked the solution (step 6) by incorporating it into the problem and seeing if the resulting story made sense.

In the CAU problems sessions, the probe consisted of one FAU problem and five CAU problems.

(3) Instruction on IAU problems: In the IAU instruction sessions, the following problems were practiced: FAU and CAU review problems, and "get less" and "get more" problems, the latter inconsistent. As in the CAU instruction, after circling the initial amount, the student was told: if the unknown is circled (example: I have some candies, I lose 3, and I end up with 12, how many did I have to start with?), since this is a get less problem, the initial amount, which is the unknown, is circled. Since there is no number in the circle, addition is required to find the solution: 3+12 = 15.

At the end of these sessions, the student independently completed a probe that consisted of one "get less" FAU problem, one "get more" CAU problem, and four IAU problems (two "get more", two "get less"). Once the student passed the probes for the IAU sessions with a score of 75%, an instruction session with all the problem types was included (see the last session of the intervention phase in Figure 3 for all three students). In this session, the schematic diagrams were deleted from the sheet. The evaluation probes for this session included six problems without schematic diagrams: two FAU, two CAU and two IAU (one "get more" and one "get less").

Generalization and maintenance. After the instruction, the generalization to an untrained setting and to two-step problems was assessed. To evaluate the generalization to an untrained setting, once the instruction was completed, the students solved a probe with six one-step problems (two FAU, two CAU and two IAU, one “get more” and one “get less” of each type) in their usual classroom, accompanied by other students and their teacher, who was not the same teacher who had provided the instruction. In another additional session, the generalization to two-step problems was assessed by means of a probe similar to the one conducted in the second baseline session, which contained eight FAU problems, with every “get more” and “get less” variant. Finally, in order to check if the acquired knowledge was maintained over time, the students completed a probe with six, one-step change problems, eight weeks after the instruction.

Reliability

All the sessions were videotaped. Interobserver reliability data were collected during the baseline, instruction generalization and maintenance phases. A graduate student in education, who did not know the hypotheses of the study, recoded 30% of the students’ performance in the evaluation probes. Interobserver agreement was calculated during each phase by dividing the number of agreements by the number of agreements plus disagreements and multiplying by 100. Interobserver reliability was 100% during baseline, 92% during instruction, 98% in the generalization probes, and 100% during maintenance. The mean interobserver reliability agreement for each student across all phases was 98% for Student A, 93% for Student B and 100% for Student C. Procedural reliability measured the instructor’s performance regarding the planned behaviors, which were: the instructor (1) provides the agreed number of problems, with the agreed amounts; (2) provides the agreed material for the session; (3) lets the students solve problems independently; (4) follows the self-instruction sheet; (5) emphasizes the key aspects of each problem type; and (6) congratulates and/or encourages the student once the problem is solved. Procedural agreement was calculated by dividing the number of observed teacher behaviors by the number of planned behaviors and multiplying by 100 for 33% of the instructional sessions. The procedural reliability was 94% for Student A, 94% for Student B and 100% for Student C.

Social Validity

As already noted, during the instruction, one of the authors conducted weekly interviews with the instructors to better coordinate and track the instructional process. During these interviews, aspects related to the use of schematic diagrams, the students' acceptance of the methodology, difficulties with specific problems, the students' behavior, the motivation they exhibited and potential ways to adapt the instruction to each student were discussed. In addition, the instructors kept a written record of relevant issues for each session.

At the end of the study, the instructors answered an open response questionnaire to assess aspects involving: (1) the effectiveness of the MSBI methodology; (2) the students' acceptance and motivation; (3) the incorporation of the methodology into their teaching practice; and (4) suggestions for improvement.

Results

The results indicate that all three participants improved their performance solving one-step addition and subtraction change word problems after following a schema-based instruction. The data collected are shown in Figures 3 and 4. In Figure 3, the horizontal axis depicts the sessions and the vertical axis depicts the participants' percentage of correct answers during the baseline, intervention, generalization and maintenance sessions. Figure 4 shows the students' performance in the group of inconsistent change word problems solved during the phases.

A visual analysis of the data presented in Figure 3 shows predictable baseline patterns for every student, with high variability between them in terms of their problem-solving skills. While in the baseline sessions student A achieved a stable success rate of between 60% and 70%, students B and C obtained results below 50%. Students B and C were introduced to intervention after a stable baseline (no more than a one-point difference between the last two consecutive baseline sessions), and once the previous student had shown improvement in his performance. This occurred after 7 baseline sessions for student B and after 9 baseline sessions for student C. As Figure 4 shows, in the inconsistent problems, all three students had percentages below 50% during the baseline. In the baseline session corresponding to two-step problems, all the students obtained percentages below 50% (0% in the case of student C) (Figure 3).

During the intervention phase, after the first intervention session, devoted to learning change stories, the three students obtained a 100% success rate (Figure 3). This performance was maintained in the following intervention session, in which all the proposed problems were of the FAU type (therefore, all consistent). These results allowed us to immediately introduce CAU problems in the third intervention session. This stage of the instruction led to a significant decrease in the performance of the three students, particularly of students B and C. This decrease coincides with the initial introduction of inconsistent problems in the intervention sessions. However, we note that all the students attained a 100% success rate in just one additional intervention session. When the IAU problems, all of them inconsistent, were introduced in the fifth intervention session, only student B showed a significant decrease in performance, but managed to obtain a score in excess of 80% in the following two sessions, as did students A and C (100% in the case of student A).

To calculate the effect size, we used the percentage of non-overlapping data PND proposed by Scruggs et al. (1987). The PND index was determined by dividing the number of data points in the intervention phase that exceeded the highest data point in the baseline phase by the total number of data points in the intervention phase, and multiplying it by 100. The PND indexes between the baseline and intervention phases were: 100% for student A (very effective), 66.7% for student B (moderately effective) and 83.3% for student C (effective).

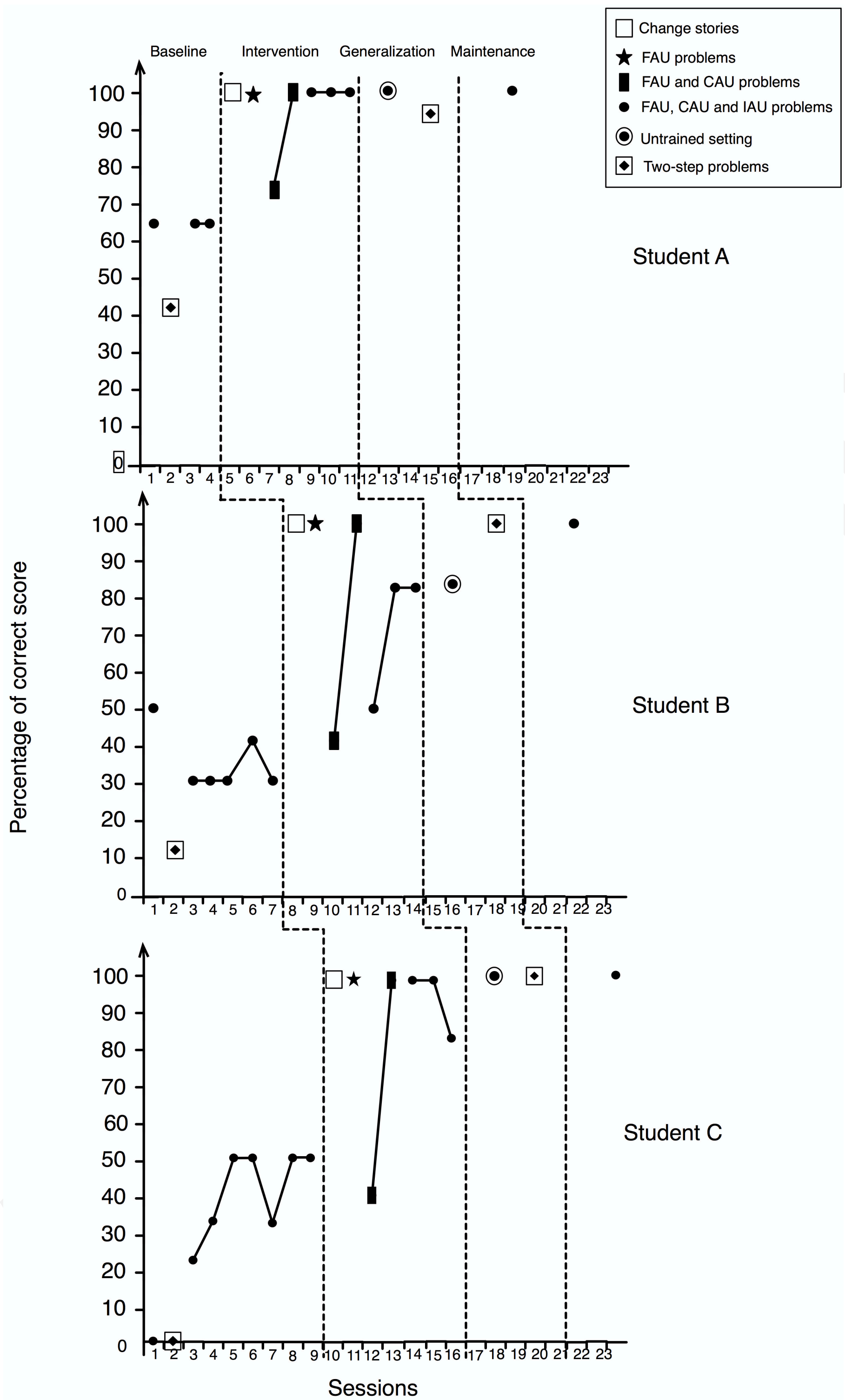


Figure 3. Percentage of the correct score during the baseline, intervention, generalization, and maintenance sessions. Note. FAU = final amount unknown; CAU = change amount unknown; IAU = initial amount unknown.

During the generalization phase, in the first generalization session, which involved solving the problems in an untrained setting, students A and C obtained 100% success and student B exceeded 80%.

In the second generalization session, devoted to solving two-step problems, two of the students achieved 100% success. The performance of the third student was also noteworthy (96%). Finally, in the maintenance session, held eight weeks after the last intervention session, all three students attained 100% scores in inconsistent problems.

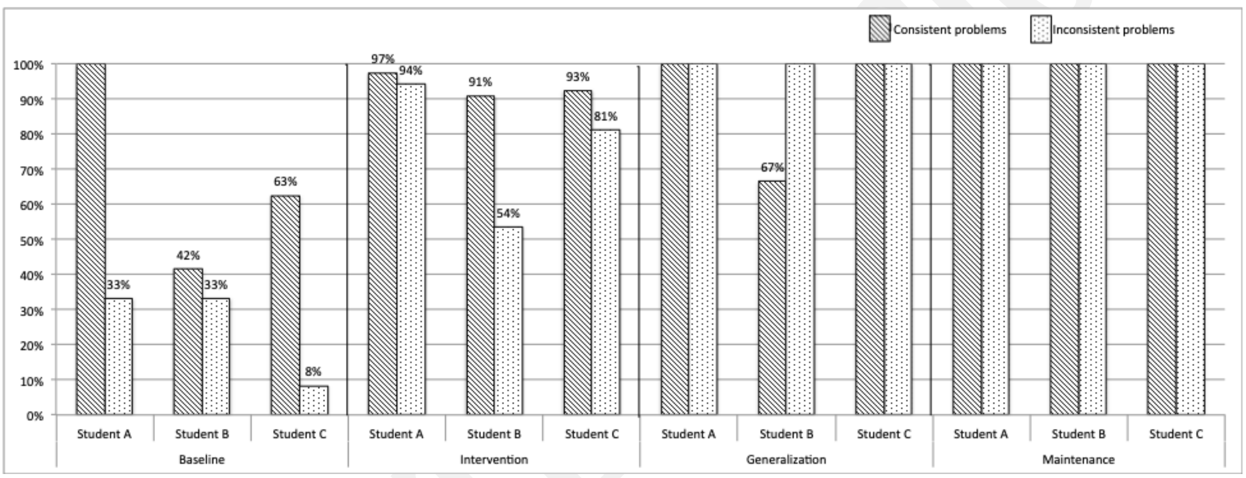


Figure 4. Percentage of correct score in consistent and inconsistent problems.

Integrating the information from all phases of the study revealed proof of the effects of the instruction at different points in time. Data show that there were important positive differences in the correct response percentages between the baseline sessions and the two first intervention sessions. After the introduction of CAU problems, the results initially worsened, followed by a significant improvement in the next intervention session. Even though each student had different prior knowledge, all three of them achieved percentages above 80% in the two last intervention sessions, in which they solved every type of addition change problem. The generalization results can be considered successful, since two of the students achieved a 100% success rate in both of the two variations introduced. Finally, the results of the maintenance phase show 100% success for the three students.

Focusing on the inconsistent change word problems (Figure 4), while in the baseline phase the three students had low success percentages with these problems (33%, 33% and 8%), their performance improved in the intervention phase (94%, 54% and 81 %), reaching 100% success in the generalization and maintenance phases.

Social Validity

The two instructors positively assessed the effectiveness of the MSBI methodology, noting the improvement observed in student performance. The two thought the methodology was suitable for use in a classroom with students with intellectual disability, and highlighted how helpful the schematic diagrams proved in helping them build a mental representation of the problem, arrange the numbers in the problem and understand the story. The instructor of student B stressed that, due to that student's problems concentrating, the use of simple schematic diagrams and of clear and brief instructions had been very beneficial. In addition, he mentioned having observed an improvement in his attitude as the instruction progressed and he began to understand the problems. The instructor of students A and C also noted the behavioral aspects, commenting that they were coming to the sessions very happy. They even opted for the instruction over other school activities that they liked and that took place at the same time. During the sessions, they seemed focused on the task and interested in doing it well. Finally, the students were clearly satisfied that they had been able to find the right answer by themselves. The instructor stated that he would use this methodology with students with similar characteristics, being careful to adapt certain aspects as needed for each student.

Discussion

The study findings showed that the schema-based approach was effective in teaching individuals with MID to solve addition and subtraction change word problems (in particular, inconsistent problems), in generalizing to two-step change problems, and in helping them retain their skills eight weeks after the instruction was completed. Although the results were positive for the three students, the specific traits of each were evidenced in their very different characteristics during the learning process, which were addressed at different times during the instructional process.

Student A only exhibited difficulties understanding the wording of inconsistent CAU problems, which led him to choose the wrong operation; he did not exhibit any problems arranging the numbers in the schema, or performing the arithmetic operation. Consequently, he did not use the concrete material and only needed to rely on the self-instruction sheet (see Figure 1), especially when inconsistent problems were introduced. Even then, it was enough for the instructor to show that the operation is not determined by the action of the verb, and to point out to him which place in the schematic diagram contained the largest amount, to help him overcome that difficulty.

Student B did not, in general, exhibit many difficulties understanding the problems. After reading the text of the problems, he often said or wrote the correct numerical solution without explicitly showing his calculation strategy; subsequently, he erroneously selected the arithmetic operation needed to solve the problem. This way of working is indicative of a student who can gain a good understanding of the problem but who ignores the arithmetic operation, and who therefore relies on calculation strategies based on counting. These strategies are not effective for large numbers and, in any case, are not helpful in furthering the search for the operation (Xin, 2019). To help him overcome these difficulties, he was asked to choose the operation (addition or subtraction) explicitly before providing the numerical solution to the problem, to carry out the selected operation and to check the answer by re-reading the problem. Carrying out these three steps on the self-instruction sheet helped the student overcome his difficulties.

Student C exhibited numerous difficulties understanding the problem in virtually every problem type, but in those cases where he could identify the operation, he was able to execute it correctly. He therefore exhibited an adequate procedural knowledge of arithmetic operations, but an inability to recognize when they are required in real situations. The challenge for this student was to help him gain an adequate understanding of the situation in the problem. In this case, and given that the student was very disciplined in following instructions, the instructor used the self-instruction sheet to solve the problems step by step and accompanied it with concrete material to model the situation.

The results obtained in this study complement those of previous research on the effectiveness of the SBI methodology for teaching problem-solving skills to students with learning difficulties. For example, in the study by Rockwell, Griffin and Jones (2011), all types of addition and subtraction problems were used in an individual with ASD, who was instructed on problems with the unknown in the final position. This was also the case in the study by Kasap and Ergenekon (2017), which delved into addition and subtraction comparison problems. The performance on change problems was specifically analyzed in the work by Desmarais et al. (2019), but they focused mainly on observing how the calculation strategies used by students to obtain a numerical solution to the problems evolved. Other studies have complemented SBI instruction with other instructional methods. For example, Jitendra et al. (2013) employed a small-group SBI tutoring intervention with third-grade students at risk for MD, demonstrating that this method conducted by trained tutors helped improve student performance. Other modified SBI approaches have been followed by Cox and Root (2020) and Browder et al. (2018). These authors have demonstrated that modified SBI – including pictorial task analysis, graphic organizers, systematic prompting with feedback, and the use of concrete and virtual manipulatives – may help to improve the performance of students with disabilities and autism.

While these approaches provide effective methods, more evidence-based mathematical practices for students with mild intellectual disability are needed in order to delve into how SBI instruction may be adapted to specific student characteristics. Our work adds to the findings of these studies that use the MSBI approach. Our main contribution lies in our detailed analysis of addition and subtraction change problems, which shows the specific instructional aspects that helped to overcome the difficulties encountered by every student, especially in the case of inconsistent problems. The gradual incorporation of these problems, and the follow-up of the steps that guided the instruction, allowed the students to improve their understanding of the situations and operations involved, despite their initial differences.

Implications for Practice

The majority of the change problems that are presented to students (with and without intellectual disability) are consistent (Parmar et al., 1996; Vicente et al., 2018). This has the result of promoting the use of direct-translation strategies, which consists of solving the problem by counting based on some keyword in the problem. These strategies circumvent the student's need to create a mental representation of the word problem structure and to have good reading comprehension (Boonen et al., 2016), causing them to make mistakes when solving inconsistent problems. The results of this work show that, through proper instruction, the participating MID students were able to improve their performance when solving change word problems, in particular, inconsistent problems.

As we have seen, the study relied on making simple adaptations for each student in relation to the instruction provided. Based on a common approach, each participant required different reinforcements and adaptations of the instruction in order to develop the desired problem-solving skills. It is essential that teachers present a sufficient variety of problems, know these teaching methods and make careful plans that are tailored to the specific needs of each student (Polo-Blanco & González, 2021; Polo-Blanco et al., 2019; 2021).

Limitations and Future Research

One of the limitations of this study is that a single case study is considered in the research, meaning the results cannot be generalized to other individuals with intellectual disability. Moreover, although the study evaluated the generalization of the effects of the instruction to the solving of two-step problems, the generalization to other types of addition and subtraction problems, such as comparison problems, could also be evaluated.

Some researchers who use an MSBI approach (Spooner et al., 2017) propose including a component that involves generalizing problem-solving skills to real-life activities. Although the acquisition of academic skills is important, in the case of students with intellectual disability and/or autism, it is essential to work on generalizing these skills to functional situations and everyday life (Bouck et al., 2018). It is therefore necessary to consider how to meaningfully transfer the academic skills gained to non-school contexts, and to conduct studies to evaluate this transfer of skills. These aspects were not assessed in the present study.

As concerns inconsistent problems, although this study provides evidence of improved performance in solving this problem type, it only evaluates difficulties with addition and subtraction problems. Future research should thus replicate the results obtained so as to evaluate the effectiveness of the schema-based methodology when solving these types of problems with other arithmetic operations.

References

Alghamdi, A., Jitendra, A., & Lein, A. E. (2020). Teaching students with mathematics disabilities to solve multiplication and division word problems: the role of schema-based instruction. *ZDM, 52*, 125-137. https://doi.org/10.1007/s11858-019-01078-0

American Association on Intellectual and Developmental Disabilities (2011). Definition of intellectual disability. Retrieved from http://aaidd.org/intellectual-disability/definition#.UyMievldWSo

Baroody, A. J. (1999). The development of basic counting, number, and arithmetic knowledge among children classified as mentally handicapped. *International Review of Research in Mental Retardation, 22,* 51-103.

Boonen, A. J. H., De Koning, B. B., Jolles, J., & van der Schoot, M. (2016). Word problem solving in contemporary math education: A plea for reading comprehension skills training. *Frontiers in Psychology: Educational Psychology*, *7*(191), 1-10.

Bouck, E. C., Park, J., Shurr, J., Bassette, L., & Whorley, A. (2018). Using the virtual–representational–abstract approach to support students with intellectual disability in mathematics. *Focus on Autism and Other Developmental Disabilities*, *33*, 237-248.

Bowman, J. A., McDonnell, J., Ryan, J. H., & Fudge-Coleman, O. (2019). Effective mathematics instruction for students with moderate and severe disabilities: a review of the literature. *Focus on Autism and Other Developmental Disabilities, 34*, 195-204.

Browder, D. M., Spooner, F., Lo, Y. Y., Saunders, A. F., Root, J. R., Ley Davis, L., & Brosh, C. R. (2018). Teaching students with moderate intellectual disability to solve word problems. *The Journal of Special Education*, *51*, 222-235.

Bruno, A., Polo-Blanco, I., González M. J., González-Sánchez, J. (2021) Strategies for solving addition problems using modified schema-based instruction in students with intellectual disabilities. Mathematics [Special Issue: Research on powerful Ideas for enriching school mathematical learning], 9(1814), 1–17. https://doi.org/10.3390/math9151814

Carpenter, T., & Moser, J. (1984). Acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education, 5*, 179-202.

Cawley, J., Parmar, R., Foley, T. E., Salmon, S., & Roy, S. (2001). Arithmetic performance of students: Implications for standards and programming. *Exceptional Children, 67*(3), 311-328.

Cox, S. K., & Root, J. R. (2020). Modified schema-based instruction to develop flexible mathematics problem-solving strategies for students with autism spectrum disorder. *Remedial and Special Education*, *41*(3), 139-151.

Daroczy, G., Wolska, M., Meurers, W. D., & Nuerk, H. C. (2015). Word problems: a review of linguistic and numerical factors contributing to their difficulty. *Frontiers in Psychology*, *6*(66), 1-13.

De Corte, E., Verschaffel, L., & De Win, L. (1985). Influence of rewording verbal problems on children's problem representations and solutions. *Journal of Educational Psychology, 77*, 460-470.

De Koning, B. B., Boonen, A. J., & van der Schoot, M. (2017). The consistency effect in word problem solving is effectively reduced through verbal instruction. *Contemporary Educational Psychology, 49*, 121-129.

Desmarais, K., Osana, H. P., & Lafay, A. (2019). Schema-based instruction: supporting children with learning difficulties and intellectual disabilities. In K. M. Robinson (Ed.), *Mathematical learning and cognition in early childhood* (pp. 203–221). Switzerland AG: Springer Nature.

Fuchs, L. S., Fuchs, D., Seethaler, P. M., Cutting, L. E., & Mancilla-Martinez, J. (2019). Connections between reading comprehension and Word-Problem solving via oral language comprehension: Implications for comorbid learning disabilities. *New Directions for Child and Adolescent Development, 165*, 73-90.

Fuchs, L. S., Powell, S. R., Seethaler, P. M., Cirino, P. T., Fletcher, J. M., Fuchs, D., Hamlett, C. L., & Zumeta, R. O. (2009). Remediating number combination and word problem deficits among students with mathematics difficulties: A randomized control trial. *Journal of Educational Psychology, 101*, 561-576.

Gevarter, C., Bryant, D. P., Bryant, B., Watkins, L., Zamora, C., & Sammarco, N. (2016). Mathematics interventions for individuals with autism spectrum disorder: A systematic review. *Review Journal of Autism and Developmental Disorders*, *3*, 224-238.

Ginsburg, H., & Baroody, A. (2007). *Test of early mathematics ability* (3rd ed.). [Spanish language version, adapted by M.C. Núñez y I. Lozano]. Madrid: TEA Ediciones.

Hegarty, M., Mayer, R. E., & Monk, C. A. (1995). Comprehension of arithmetic word problems: A comparison of successful and unsuccessful problem solvers. *Journal of Educational Psychology, 87*(1), 18-32. https://doi.org/10.1037/0022-0663.87.1.18

Hord, C., & Bouck, E. C. (2012). Review of academic mathematics instruction for students with mild intellectual disabilities. *Education and Training in Autism and Developmental Disabilities, 47*, 389-400.

Jitendra, A. K., Hoff, K., & Beck, M. M. (1999). Teaching middle school students with learning disabilities to solve word problems using a schema-based approach. *Remedial and Special Education, 20*, 50-64.

Jitendra, A., DiPipi, C. M., & Perron-Jones, N. (2002). An exploratory study of schema-based word problem solving instruction for middle school students with learning disabilities: An emphasis on conceptual and procedural understanding. *Journal of Special Education*, *36*, 23-38.

Jitendra, A. K., Dupuis, D. N., Rodriguez, M. C., Zaslofsky, A. F., Slater, S., Cozine-Corroy, K., & Church, C. (2013). A randomized controlled trial of the impact of schema-based instruction on mathematical outcomes for third-grade students with mathematics difficulties. *The Elementary School Journal*, *114*, 252-276.

Kasap, C., & Ergenekon, Y. (2017). Effects of a schema approach for the achievement of the verbal mathematics problem-solving skills in individuals with autism spectrum disorders. *Educational Sciences: Theory & Practice, 17*, 1787-1809.

Lean, G. A., Clements, M. A., & Del Campo, G. (1990). Linguistic and pedagogical factors affecting children's understanding of arithmetic word problems: A comparative study. *Educational Studies in Mathematics, 21*(2), 165-191.

Lewis, A. B., & Mayer, R. E. (1987). Students' miscomprehension of relational statements in arithmetic word problems. *Journal of Educational Psychology*, *79*, 363-371.

Pape, S. J. (2003). Compare word problems: Consistency hypothesis revisited. *Contemporary Educational Psychology, 28*, 396–421. http://dx.doi.org/10.1016/S0361-476X(02)00046-2.

Parmar, R. S., Cawley, J. F., & Frazita, R. R. (1996). World problem-solving by students with and without mild disabilities. *Exceptional Children, 62*, 415-429.

Polo-Blanco, I., González, M. J., & Bruno, A. (2019). An exploratory study on strategies and errors of a student with autism spectrum disorder when solving partitive division problems. *Brazilian Journal of Special Education, 25*(2), 247–264. https://doi.org/10.1590/s1413-65382519000200005

Polo-Blanco, I., González, E. M. (2021) Teaching addition strategies to students with learning difficulties. *Autism & Developmental Language Impairments, 6*, 1–14. https://doi.org/10.1177/23969415211045324

Polo-Blanco, I., González, M. J., & Bruno, A. (2021). Influencia del contexto en problemas de multiplicación y división: estudio de caso de un alumno con autismo. [Influence of the context on multiplication and division problems: A case study of a student with autism]. Siglo Cero, 52(1), 59–78. https://doi.org/10.14201/scero20215215978

Polo-Blanco, I., Van Vaerenbergh, S., Bruno, A., & González, M. J. (2022). Conceptual model-based approach to teaching multiplication and division word-problem solving to a student with autism spectrum disorder. *Education and Training in Autism and Developmental Disabilities, 57*(1), 31-43.

Riley, M. S., Greeno, J. G., & Heller, J. L. (1983). Development of children's problem-solving ability in arithmetic. In H. P. Ginsburg (Ed.), *The development of mathematical thinking* (pp. 153-196). New York: Academic Press.

Rockwell, S. B., Griffin, C. C., & Jones, H. A. (2011). Schema-based strategy instruction in mathematics and the word problem-solving performance of a student with autism. *Focus on Autism & Other Developmental Disabilities, 26*, 87-95.

Root, J. R., Browder, D. M., Saunders, A. F., & Lo, Y. (2016). Schema-based instruction with concrete and virtual manipulatives to teach problem solving to students with autism. *Remedial and Special Education, 48,* 42-52.

Schopler, E., Reichler, R. J., & Renner, B. R. (1988). *Childhood Autism Rating Scale.* Los Angeles: Western Psychological Services.

Schumacher, R. F., & Fuchs, L. S. (2012). Does understanding relational terminology mediate effects of intervention on compare word problem? *Journal of Experimental Child Psychology, 111*, 607-628. http://dx.doi.org/10.1016/j.jecp.2011.12.001.

Scruggs, T. E., Mastropieri, M. A., & Casto, G. (1987). The quantitative synthesis of single-subject research: Methodology and validation. *Remedial and Special Education, 8,* 24–33.

Spooner, F., Saunders, A., Root, J., & Brosh, C. (2017). Promoting access to common core mathematics for students with severe disabilities through mathematical problem solving. Research & Practice for Persons with Severe Disability, 42, 171-186. doi:10.1177/1540796917697119

Van der Schoot, M., Bakker-Arkema, A. H., Horsley, T. M., & Van Lieshout, E. C. D. M. (2009). The consistency effect depends on markedness in less successful but not successful problem solvers: An eye movement study in primary school children. *Contemporary Educational Psychology, 34*, 58-66. http://dx.doi.org/10.1016/j.cedpsych.2008.07.002.

Vicente, S., Manchado, E., & Verschaffel, L. (2018). Solving arithmetic word problems. An analysis of Spanish textbooks. *Cultura y Educación*, *30*, 71-104.

Wong, C., Odom, S. L., Hume, K., Cox, A. W., Fettig, A., Kucharczyk, S., Brock, M.E., Plavnick, J.B., Fleury, V.P., &, Schultz, T. R. (2015). Evidence-based practices for children, youth and young adults with autism spectrum disorder. *Journal of Autism and Developmental Disorders, 45*, 1951-1966. doi:10.1007/s10803-014-2351-z

Xin, Y. P. (2008). The effect of schema-based instruction in solving word problems: An emphasis on pre-algebraic conceptualization of multiplicative relations. *Journal for Research in Mathematics Education, 39*, 526–551.

Xin, Y. P. (2019). The effect of a conceptual model-based approach on "additive" word problem solving of elementary students who are struggling in mathematics. *ZDM: Mathematics Education*, *51*, 139-150.

Xin, Y. P., & Jitendra, A. K. (1999). The effects of instruction in solving mathematical word problems for students with learning problems: A meta-analysis. *The Journal of Special Education, 32,* 207-225.

---

[1] Pictographic symbols adapted from ARASAAC (http://arasaac.org), created by Sergio Palao and distributed under the Creative Commons license (BY-NC-SA).