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Representations and generalization in early algebra: a comparative study of autistic students and their non-autistic peers

Irene Polo-Blanco[1] · Maria Chimoni[2] · Juncal Goñi-Cervera[1] · Demetra Pitta-Pantazi[2]

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Abstract
This study explores ASD (Autism Spectrum Disorder) students' early algebraic thinking abilities by comparing them to their non-ASD peers. The first aim was to examine whether possible significant differences between ASD and non-ASD students in arithmetic also extend to early algebra. The second aim focused further on early algebraic thinking, examining whether ASD students differ from their non-ASD peers in the modes of representation they use (concrete, figural, arithmetical, and symbolic) and the levels of generalization they achieve (factual, contextual, and symbolic) when engaging with early algebra tasks. Using a mixed-methods approach, we analyzed data from 26 ASD and 26 non-ASD students aged from 6 to 12 years old. Statistical analyses revealed that while ASD students' performance in the arithmetic test was lower than non-ASD students, their performance on the early algebra test was comparable. For ASD students, the figural mode of representation was a significant predictor of their total score in the early algebra test. For non-ASD students, no specific mode of representation significantly predicted their total score in the early algebra test. At the same time, factual generalizations were a significant predictor of ASD students' total scores in the early algebra test, whereas contextual and symbolic generalizations were significant predictors for non-ASD students. These findings suggest that while ASD and non-ASD students achieve similar total scores on the early algebra test, they differ in their use of specific modes of representation and the level of generalization they attain. ASD students seem to benefit from creating figural representations and tend to achieve more basic levels of generalization, compared to their non-ASD peers, who demonstrate greater flexibility in using various modes of representation and reach more advanced levels of generalization.

Keywords Autism Spectrum Disorder (ASD) · Early algebraic thinking · Patterns · Representations · Generalization

✉ Irene Polo-Blanco
irene.polo@unican.es

[1] Universidad de Cantabria, Santander, Spain

[2] University of Cyprus, Nicosia, Cyprus

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1 Introduction

In recent decades, researchers, policymakers, and curriculum designers worldwide have emphasized the importance of understanding how various student populations interact with school mathematics, recognizing that a one-size-fits-all approach is inadequate. Such recommendations stem from the growing recognition of the unique cognitive profiles of students with learning difficulties, which can affect their mathematical learning and the need for developing instructional strategies that address diverse learning needs (e.g., Faragher et al., 2016; Gutierrez, 2011; Leonard, 2018).

The current study focuses on students with Autism Spectrum Disorder (ASD) and their non-ASD peers. Previous research highlights a distinct cognitive profile in students with ASD compared to their non-ASD peers (Mayes & Calhoun, 2006; Wilson, 2024), influencing their mathematical performance (Mayes & Calhoun, 2006; Tonizzi & Usai, 2024). For example, students with ASD may show weaknesses in language, working memory, and theory of mind (understanding that others have thoughts, beliefs, and emotions different from one's own) but demonstrate strengths in visuospatial abilities, all of which are linked to their arithmetic and problem-solving performance (Baron-Cohen et al., 2000; Chen et al., 2019; Fernández-Cobos et al., 2025; Polo-Blanco et al., 2024).

Despite these findings, there remains a significant lack of understanding of how these characteristics may extend to other areas of mathematics, such as algebra. This is particularly critical as early algebra is an integral part of most contemporary mathematics curricula in the primary grades, making it an area that all students encounter from the start of their mathematical learning (Kieran, 2020). Moreover, an increasing number of students with ASD are entering secondary education, where they are expected to achieve proficiency in the same mathematics curriculum content, including algebra, as their typically developing peers (Barnet & Cleary, 2019).

In this context, a research study examining early algebraic thinking in students with ASD is important for at least two key reasons. First, early algebraic thinking is a crucial component of mathematics education, laying the foundation for more advanced mathematical concepts and procedures in later stages (Cai & Knuth, 2005). Second, there is limited research on how students with ASD perform on early algebra tasks compared to their non-ASD peers. Previous research has primarily focused on evaluating teaching methodologies for students with ASD in algebraic problem-solving (Root & Browder, 2019), equation-solving (Barnet & Cleary, 2019), and generalization tasks (Goñi-Cervera et al., 2024). However, only a few studies have examined their performance in early algebraic tasks (e.g., Goñi-Cervera et al., 2022), while this has been widely documented for typically developing students (e.g., Radford, 2003; Stephens et al., 2017). This highlights a critical research gap regarding the specific challenges or strengths that students with ASD may encounter compared to their non-ASD peers when working on foundational early algebra tasks.

This study aims to address this gap by investigating potential performance differences between ASD and non-ASD students on early algebra. To this end, the mean scores of ASD and non-ASD students on arithmetic and early algebra tests are compared to primarily determine whether potential performance differences between the two groups are similar in both areas. Focusing further on early algebra, this study also explores ASD and non-ASD students' approaches, examining the mode of representation they use to express a relationship and the sophistication level of the generalization they achieve. In this way, the study seeks to deepen our understanding of how students with ASD engage with early algebra tasks compared to their non-ASD peers and to inform the development of more effective educational strategies tailored to their needs.

2 Literature review

2.1 Autism, mathematical learning, and early algebraic thinking

ASD is a chronic neurodevelopmental disorder characterized by difficulties in communication and social interaction, along with repetitive patterns of behavior and restrictive interests (DSM-5; American Psychiatric Association, 2013). Individuals with ASD exhibit a wide range of symptoms and are highly diverse in their cognitive capabilities, with some displaying strengths in certain areas while facing challenges in others (Hill, 2004). Recent data indicate an increase in the prevalence of ASD, suggesting that one in every 100 children worldwide is diagnosed with autism (Zeidan et al., 2022).

Previous research has shown that students with ASD of varying ages experience more learning difficulties in different mathematical concepts compared to their typically developing peers (Aagten-Murphy et al., 2015; Bullen et al., 2022; Dowker, 2020; Fernández-Cobos et al., 2025; Mayes & Calhoun, 2006; Polo-Blanco et al., 2024). Some of these studies have focused on early mathematical proficiency (Fernández-Cobos & Polo-Blanco, 2024), while others have focused on word problem-solving abilities (Bullen et al., 2022; Polo-Blanco et al., 2024). Overall, these results suggest that the characteristics of the disorder may contribute to the mathematical difficulties observed among students with ASD. For example, the performance of students with ASD in word problem-solving may be affected by difficulties with language comprehension (Bae et al., 2015) or a low theory of mind profile (Polo-Blanco et al., 2024). Specific characteristics of ASD are also believed to significantly impair the development of algebraic thinking (Barnet & Cleary, 2019). For instance, individuals with ASD often exhibit concrete thinking, which can lead to difficulties with generalization and abstract symbolic reasoning (Minshew et al., 2002). At the same time, strengths such as a predilection for visual thinking—described by Grandin (1995) as the ability to process information and formulate reasoning through mental imagery and visual systems—have been found to benefit students with ASD. According to Sahyoun et al. (2010), typically developing students often rely on linguistically mediated pathways for thinking, whereas autistic cognition tends to engage more with visuospatial processing networks. This visual thinking frequently manifests in the use of drawings which may serve as a primary channel of expression given the communication challenges associated with autism (Bruno et al., 2024; Di Renzo et al., 2017; Polo-Blanco et al., 2024).

The predilection for visual thinking among students with ASD can also be beneficial in algebraic tasks that involve figural representations or pattern recognition (Barnet & Cleary, 2019). A previous study by Goñi-Cervera et al. (2022) showed that ASD students who engaged with a figural pattern task relied strongly on drawings, especially for identifying near terms of the pattern. Those who were successful in generalizing utilized a combination of drawing and arithmetical approaches. Additionally, a recent study by Goñi-Cervera et al. (2024) found that explicit instruction using multiple representations and mediation improved the performance of students with ASD on generalization tasks.

To deepen our understanding of the abilities of ASD students, it is essential to compare the performance of ASD students in an early algebra context with that of typically developing students of similar ages. However, considering that students with ASD are highly diverse in their cognitive capabilities, with variability in the severity and manifestation of these features (Hill, 2004), any results obtained should be interpreted with caution as potential performance differences between ASD and non-ASD students in early algebra may be attributed to these and other factors, such as differences in arithmetic abilities.

The following section is focused on early algebraic thinking as it has been explored through theoretical frameworks and research findings with typically developing students. This examination establishes a foundation for defining important characteristics of early algebraic thinking, which can then be assessed among students with ASD.

2.2 Early algebraic thinking

Various frameworks over the last 20 years have been developed to outline the basic components of algebraic thinking (e.g., Blanton et al., 2015; Chimoni et al., 2018; Kaput, 2008; Kieran, 2007). A common thread among them is the process of generalization (Kaput, 2008; Kieran, 2022). Generalization refers to identifying and expressing a general relationship derived from a set of specific instances (Blanton et al., 2015), while it is rooted in the ability to recognize "the same and the different" (Radford, 2008, p. 83). Different semiotic signs can be used for expressing generalizations, including gestures, language, drawings, diagrams, tables, and the alphanumeric symbols of algebra (Radford, 2000). As students advance in their education, generalizations are expected to be expressed with conventional symbol systems (Kaput, 2008).

The process of generalization is studied in the current study through functional thinking, one of the core algebra content strands, which pertains to the mathematical concept of functions (Kaput, 2008). Functional thinking is often viewed as a pathway into early algebra, particularly via figural pattern tasks (Blanton & Kaput, 2011; Radford, 2006; Wilkie & Clarke, 2016). Figural pattern tasks typically require students to extract the relationship between a term and its position and use this relationship to determine terms in other positions. As such, they help students formulate generalizations about the relationship between two quantities in ways that other functional thinking tasks may not (Rivera, 2010). For example, in the figural pattern illustrated in Fig. 1, the general relationship between a term (y) and its position (x) is represented by the equation $y = 4x + 1$.

Smith (2008) proposed three ways for analyzing figural pattern tasks: (a) "recursive patterning" as the identification of variation within a single sequence of values (e.g., adding 4 to find the next term), (b) "correspondence thinking" as recognizing how two quantities are

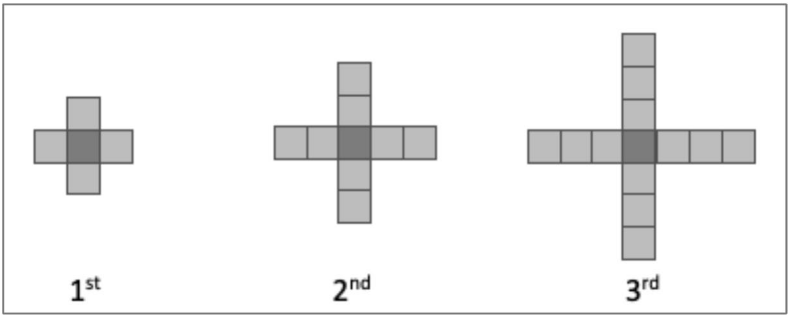

Fig. 1 Example of a figural pattern

correlated (e.g., y is always 4 times x plus 1), and (c) "covariational thinking" as the examination of how two quantities vary simultaneously (e.g., "as x increases by one, y increases by four"). The conceptual understanding of function involves analyzing this covariational relationship between two changing quantities (Confrey & Smith, 1995), recognizing that "every value of one quantity determines exactly one value of the other" and that this relationship remains invariant across the values of both quantities (Thompson & Carlson, 2017, p. 436).

Prior research studies indicated that students in the early grades usually follow a path from recursive patterning to correspondence thinking, which involves levels from no generalization to quasi-generalization to formal generalization (Cooper & Warren, 2011). Radford (2003) provided additional evidence for three distinct types of generalizations achieved through the coordination of different semiotic means: the factual, the contextual, and the symbolic. A factual generalization occurs when students use concrete actions to find a particular term in a sequence (e.g., Fig. 10 equals Fig. 9 plus 4). This level often involves additive relationships and phrases as "the next" or "always" (e.g., you always add 4 to find the next term). A contextual generalization is evident when students refer to the spatial characteristics of a term, using language to describe the generalization (e.g., the 10 th term has one square in the middle and 4 branches with 10 squares each). A general term could also be expressed in the same way (e.g., there is always a square in the middle and four branches with squares equal to the term's number). Finally, a symbolic generalization involves using alphanumeric symbols to express a general relationship without spatial explanations (e.g., $y = 4x + 1$).

Additionally, empirical findings confirm that students may use various modes of representation to express quantitative relationships in figural patterns (e.g., Blanton et al., 2015; Chimoni et al., 2018). Some students use drawings for factual generalizations, while others use verbal expressions or alphanumeric symbols to represent the relationship between quantities, reflecting contextual or symbolic generalizations (Chimoni et al., 2018). It is important to note that the same figural pattern task may be solved correctly by students who used different modes of representation and reached different levels of generalization (Blanton et al., 2015; Chimoni et al., 2018). For example, a student may find the general term of a pattern using contextual generalization, rather than using an abstract symbolic generalization that is detached from the context. While the final answer may be the same, the process behind arriving at that answer can differ.

Previous studies also showed that instruction can improve students' ability to generalize, especially by helping them recognize spatial structures and their connection to numerical aspects (Mason et al., 2009; Warren et al., 2016). Concrete materials, such as building blocks, are also been found to aid this process (Carraher et al., 2008; Goñi-Cervera et al., 2022; Twohill, 2018).

Summing up, numerous studies suggest variations in levels of generalization and modes of representation among students (Chimoni et al., 2018). Most of these studies have focused on typically developing students, with limited research on comparisons between different student populations. However, teachers often work in mixed-ability classrooms, highlighting the importance of exploring these aspects in students with varying profiles, including those with ASD.

3 The current study

3.1 Aim and hypotheses of the study

This study aims to explore ASD students' early algebraic thinking abilities. To achieve this, we first compare the total score of ASD students with their non-ASD peers on both an early algebra test and an arithmetic test. These comparisons will clarify whether documented arithmetic differences between the two groups, also exist in algebra. Second, we concentrate on the early algebra test and record, the modes of representation that the students used and the levels of generalization they achieved. This focus is expected to clarify how students with ASD engage with early algebra tasks and differ from their non-autistic peers.

Building upon previous investigations into early mathematical abilities in autism (Fernández-Cobos & Polo-Blanco, 2024; Fernández-Cobos et al., 2025; Goñi-Cervera et al., 2022; Polo-Blanco et al., 2024), we hypothesize that ASD students may demonstrate lower success rates on figural pattern tasks compared to their non-autistic peers. Concerning modes of representation, given the visual-thinking style characteristic of autism, we anticipate a higher prevalence of drawing-based representations among autistic students. Additionally, due to the concrete thinking style and challenges with generalization (Minshew et al., 2002), we expect that autistic students will encounter greater difficulties with symbolic generalization than their non-autistic counterparts.

3.2 Research questions

The study's research questions are the following:

(a) Is there a statistically significant difference in the mean performance of ASD and non-ASD students in the early algebra and arithmetic tests?
(b) What modes of representation and levels of generalization predict ASD students' performance on early algebra tasks? Is this predictive model the same as for non-ASD students?

4 Method

4.1 Study design

The study had an exploratory design and used a mixed approach that involved both qualitative and quantitative methods for collecting and analyzing the data.

4.2 Participants

The study was conducted with 26 students in the ASD group (23 males and 3 females, mean age 9.35) and 26 students in the non-ASD group (23 males and 3 females, mean age 9.41), with ages ranging from 6 to 12 years old covering the entire period of primary education in Spain. Autism is identified in a relatively small percentage of the population, making it challenging to collect a sufficiently large and representative sample from a single age cohort. For this purpose, several studies involved participants with autism across different age groups to address small sample sizes or the need for developmental insight (e.g., Lord et al., 2012; Pellicano et al., 2014). The ASD participants were recruited from various health, social, and educational resources serving individuals with autism in the Spanish region of Cantabria. Recruitment occurred between July 2019 and February 2021. The selected ASD participants attended 19 different schools. For each ASD participant, a non-ASD counterpart of the same sex, age, school, grade, and classroom was matched with the assistance of school counselors or managers. All participants had intellectual disabilities ruled out, with IQ scores of 70 or higher, as assessed using the WISC-V (Weschler, 2015).

Parents or legal guardians received a detailed explanation of the study's purpose and procedures before providing written informed consent. They were informed that the child's anonymity would be safeguarded and that their child could withdraw from the study at any point if they wished to. The Cantabria Research Ethics Committee (CEIC) approved this study.

4.3 Data collection process

This study is part of a larger project that assessed mathematical abilities —including arithmetical thinking, mathematical problem-solving, and algebraic thinking —and their relationship to cognitive variables. The team that participated in the larger project consisted of psychologists, psychiatrists, mathematics education researchers (including the first and third authors), and active teachers from different educational levels. A psychologist conducted two or three sessions with each participant to assess cognitive variables. Additionally, two or three sessions, led by a mathematics education researcher, were held to administer performance tests evaluating students' mathematical abilities: (a) an arithmetical test (Test on Early Mathematical Abilities, TEMA-3; Ginsburg & Baroody, 2007), (b) a test containing mathematical problems and (c) an early algebra test (described below). All sessions were videotaped and transcribed for subsequent analysis.

In a previous article (Polo-Blanco et al., 2024), the results concerning the relationship between cognitive abilities and mathematical problem-solving were presented. One of the key findings was that a higher proportion of ASD students exhibited difficulties in mathematical problem-solving, particularly those with lower cognitive abilities (e.g., poor inhibition, verbal comprehension, and theory of mind). These results confirm the variability in the cognitive and mathematical abilities of ASD students and highlight that autism cannot solely be used as a single measure to explain differences in student performance. The current article focuses on comparing the performance of these students and their non-ASD peers on the arithmetical test and the early algebra test. Further, it analyzes students' responses to the early algebra tasks to uncover qualitative characteristics of their approach.

4.4 Instruments and scoring

4.4.1 Early algebra test

A test was designed in written format to assess students' early algebraic thinking. The test featured a figural pattern task adapted from Carraher et al. (2008), involving the function $f(x) = 2x + 2$. This task has been utilized in previous studies with elementary students of similar age ranges (e.g., Blanton et al., 2015) and aligns with the focus of the revised Spanish curriculum LOMLOE (2022) on algebraic sense as fundamental knowledge.

The early algebra test was administered through individual semi-structured interviews, allowing students the option to respond with written or verbal answers. The interviewer introduced the task by presenting scenarios with one and two tables (see Fig. 2). The task involved six questions that addressed specific terms (e.g., “If we join three/four/five/eight/18/100 tables, how many people can sit around them?”) as well as one addressing the general term (e.g., “If you know how many tables there are, how can you figure out the number of people who can sit around them?”). The interviewer gave the participants a paper test, a pen, and interlocking building blocks. If necessary, the interviewer assisted in reading the task and encouraged the students to find solutions. For each question, students were asked to explain their thinking process.

As described in the literature review, the correct answers to these questions could be reached by using different modes of representation and varying levels of generalization. For example, the question about the next term or other consecutive terms of the pattern could be answered by either continuing the drawing, identifying the arithmetic recursive pattern (adding two), or directly recognizing the correspondence relationship between the position of the term and the term itself. Similarly, the generalization could be either factual, based on specific drawings, concrete representations, and arithmetic additions, contextual or symbolic.

We followed a two-step procedure to record students’ responses to this test. The first step was to score the response to correct or incorrect, irrespective of the student’s approach. The total score for the early algebra test ranged from 1 to 7, with one point awarded for a correct answer and zero points for an incorrect answer. The scale’s internal consistency was evaluated using Cronbach’s alpha, and its value was $\alpha = .86$, indicating satisfactory reliability. The student’s total score on this test will be referred to as “performance” throughout the article.

The second step was to identify the mode of representation and the level of generalization exhibited by the students in each of their responses. Based on the videos and transcripts, the frequency of the different modes of representation used and the levels of generalization expressed while tackling each question of the early algebra test were recorded. To ensure consistent coding of students’ responses to the tasks, we created a codebook of

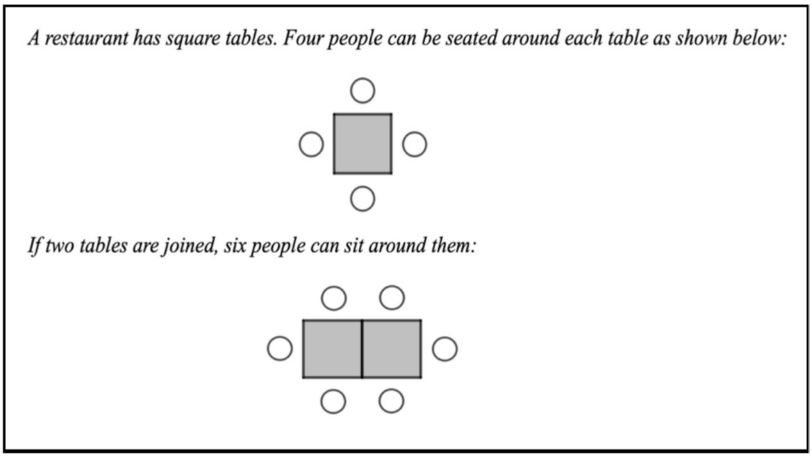

Fig. 2 The pattern task

Table 1 The framework for analyzing students' modes of representation and levels of generalization

Mode of representation	Concrete materials (e.g., the interlocking cubes) Figural (e.g., drawings) Arithmetical (e.g., repeated addition) Algebraic (verbal expressions or alphanumeric symbols)
Levels of generalization	Factual (e.g., using particular instances) Contextual (e.g., referring to the spatial–temporal characteristics of the objects in the pattern) Symbolic (e.g., using alphanumeric symbols to express the generalization and not including spatial explanations)

examples of the different expected modes of representations and levels of generalizations based on the literature review, thus applying a deductive procedure (Mayring, 2015).

Two researchers independently coded 40% of the student responses using the framework presented in Table 1. The kappa coefficient for inter-rater agreement was calculated for all the responses, with values ranging from 0.85 to 1, indicating substantial to perfect agreement. All the disagreements between the raters were discussed among all researchers until a common consensus was reached.

On occasions where a student used two modes of representation to solve the same question, the more advanced mode of representation was coded. For example, in the case of four tables, a student might use a figural representation by drawing the tables and the people around them, followed by an arithmetic representation, such as writing 4+ 4+ 2. In this case, the arithmetical mode was coded.

4.4.2 Arithmetic test (TEMA-3)

Arithmetical abilities were evaluated using the "Early mathematical abilities TEMA-3 test" (Ginsburg & Baroody, 2007), specifically designed to assess mathematical knowledge in children. This performance test comprises 72 items, evaluating counting, comparing numbers, mastery of number facts, and calculation skills.

Test scores ranged from 0 to 72 and were then converted into a mathematical age. The internal consistency of the test has been reported at 0.90 for the neurotypical population (Ginsburg & Baroody, 2007) and has been used in prior research involving children with ASD (Fernández-Cobos & Polo-Blanco, 2024; Fernández-Cobos et al., 2025; Polo-Blanco et al., 2024). For the current study, the scale's internal consistency was evaluated using Cronbach's alpha, which yielded a value of $\alpha = 0.92$, indicating excellent reliability.

4.4.3 Data analysis

To address the first research question, we conducted a Multiple Analysis of Variance (MANOVA). The dependent variables were students' total scores on the early algebra test and the arithmetic test, while the fixed factor was their group, either ASD or non-ASD. Thus, this analysis assessed significant mean score differences in early algebra and arithmetic between the two groups.

For the second research question, two Ordinal Regression Analyses were conducted for each group of students. In the first analysis, the independent variable was the modes of representation (i.e., concrete, figural, arithmetic, algebraic). In the second analysis, the interdependent variable was the levels of generalization (e.g., factual, concrete, contextual, symbolic). These variables were measured based on their frequency across all tasks attempted by each student. In both analyses, the dependent variable was students' total score on the early algebra test, categorized into ordinal levels (i.e., low, medium, and high performance).

Overall, these analyses aimed to determine whether specific modes of representation and levels of generalization predict higher performance levels and whether the patterns were consistent across the two groups. All statistical analyses were executed using SPSS version 29.

To gain deeper insight into the relationship between students' performance on the early algebra test and the use of different modes of representation and levels of generalization by ASD and non-ASD students, we also conducted a qualitative analysis of students' responses to the test. To achieve the purpose of the qualitative study, we employed the purposive sampling method to select one student from the ASD group and one from the non-ASD group. Purposive sampling is a common non-probability sampling technique that focuses on selecting participants who can provide the most relevant information related to the research aim (Patton, 1990). Therefore, we purposefully selected one ASD and one non-ASD student whose responses were sufficiently rich to illustrate the use of specific modes of representation and levels of generalization.

5 Results

5.1 The difference in the mean performance of ASD and non-ASD students in the arithmetical test (TEMA-3 test) and the early algebra test

Considering the first research question of whether there are statistically significant differences in the mean performance of the ASD and non-ASD students in the arithmetical test and the early algebra test, the results of the MANOVA indicated a statistically significant difference in the arithmetical test between ASD and non-ASD students ($F = 8.34$, $p =$ 0.01). Still, surprisingly, there was no statistically significant difference in the early algebra test between ASD and non-ASD students ($F = 1.38$, $p = 0.25$). This outcome was unexpected, as we had hypothesized that ASD students would exhibit lower performance in the early algebra test compared to their non-ASD peers (Table 2)[1].

Table 2 MANOVA of the early mathematical abilities tema-3 test and the early algebra test

		Sum of Squares	*df*	Mean Square	*F*	*p*
Arithmetical—Early mathematical abilities test (TEMA-3 test)	Between Groups	8.69	1	8.69	8.34	0.01
	Within Groups	52.13	50	1.04		
	Total	60.82	51			
Early algebra test	Between Groups	7.69	1	7.69	1.38	0.25
	Within Groups	279.00	50	5.58		
	Total	286.69	51			

[1] The anonymized dataset supporting the findings of this study is available in the supplementary material.

5.2 Predicting variables of ASD and non-ASD students' performance in the early algebra test

To answer the second research question which focused on the early algebra test and aimed to understand how the modes of representation and levels of generalization predict ASD and non-ASD students' performance in the early algebra test, we followed two steps. First, two linear ordinal regression analyses were conducted using the entire sample, which included both ASD and non-ASD students. This was essential to gain an overview of how well the ordinal regression models fit the data as a whole and identify any issues in the models that may affect subsequent subgroup analyses. Moreover, this enabled checking the assumptions of the statistical analysis to ensure these assumptions were not violated before proceeding to subgroup analyses.

Specifically, we checked the assumptions for linearity, independence of errors, homoscedasticity, and normality of errors. Our data met all the requirements of these assumptions. We also employed the bootstrapping method (Preacher & Hayes, 2008), which is more robust for small sample sizes, making it an appropriate choice for our study's context, given the relatively small sample size, mainly due to the inclusion of ASD students—a group that typically comprises a limited population. This statistical method allows one to generate percentile-based confidence intervals for indirect effects by a simulating resampling of the original dataset. In the current study, we simulated a resampling of 1000 iterations. Estimating confidence intervals in this manner allows for asymmetric intervals (above and below the mean estimate), thereby relaxing the assumption of multivariate normality. In both models, the dependent variable was students' performance on the early algebra test. For the first model (Mode of representation for all students), the independent variables were the frequency of concrete, figural, arithmetical, and symbolic representations. For the second model (Level of generalization for all students), the independent variables were the frequency of factual, contextual, and symbolic responses, which denoted the level of generalization.

To ensure that performance on the early algebra test is independent of the predictor variables, we conducted an additional statistical test to assess potential multicollinearity or bias between the predictor variables and the performance measure. Variance Inflation Factors (VIFs) were calculated for all predictor variables. All the VIF values were close to 1, indicating a very low level of multicollinearity among the predictors included in the regression model. Since VIF values below 5 (and especially below 10) are generally considered acceptable, this suggests that multicollinearity is not a concern for the variables in this analysis.

Table 3 presents the model fitting information and the pseudo $R^2_{\text{Nagelkerke}}$ values for the two models. The values suggest that both models explain the variation in students' performance in a statistically significant way. To put it in simple words, the modes of representation ($x^2 = 12.27, df = 5, p = 0.03$) and the levels of generalizations ($x^2 = 30.26, df = 4, p <$ 0.00) utilized by all students in our sample (both ASD and non-ASD) can predict how successful the students will be in providing correct answers to the questions of the early algebra test. The pseudo $R^2_{\text{Nagelkerke}}$ values indicate a good relationship between the predictors, modes of representations ($R^2_{\text{Nagelkerke}}$=0.22), and generalization levels ($R^2_{\text{Nagelkerke}} = 0.45$) with students' performance in the early algebra test.

Table 3 Model fitting information for level of generalization and mode of representation for all students

Dependent variable	2 Log Likelihood	Chi-Square	*df*	*p*	Pseudo $R^2_{\text{Nagelkerke}}$
Mode of representation for all students	158.63	12.27	5	0.03	0.22
Level of generalization for all students	120.28	30.26	4	0.00	0.45

Table 4 Ordinal regression analysis for the relationship between the levels of generalization and early algebra performance for all students

		Parameter estimates						
				95% Wald confidence interval		Hypothesis test		
	Parameter	Estimate	*SE*	Lower	Upper	Wald	*df*	*p*
Modes of representation	Concrete	0.50	0.24	0.03	0.98	4.39	1	0.04
	Figural	0.44	0.17	0.11	0.77	6.89	1	0.01
	Arithmetical	0.11	0.21	−0.30	0.52	0.28	1	0.59
	Algebraic	0.25	0.17	−0.10	0.59	1.99	1	0.16
Level of generalization	Factual	0.27	0.16	−0.05	0.59	2.71	1	0.10
	Contextual	0.49	0.19	0.11	0.86	6.44	1	0.01
	Symbolic	1.50	0.38	0.76	2.24	15.86	1	0.00

The ordinal regression results shown in Table 4 indicate that in terms of individual predictors, concrete (Estimate $=0.50$, $SE=0.24$, Wald $=4.39$, $p=0.04$) and figural (Estimate $=0.44$, $SE=0.17$, Wald $=6.89$, $p=0.01$) modes of representations predict students' performance in the early algebra test in a statistically significant way. This suggests that higher frequencies of using concrete and figural representations are associated with higher performance in the early algebra test. Considering the generalization levels, it was found that the contextual (Estimate $=0.49$, $SE=0.19$, Wald $=6.44$, $p=0.01$) and the symbolic (Estimate $=1.50$, $SE=0.38$, Wald $=15.86$, $p<0.00$) levels of generalization were statistically significant predictors of students' performance in the early algebra test. This suggests that higher frequencies of contextual and symbolic generalization levels are associated with higher performance in the early algebra test.

To explore whether these models are similar when the sample is split into ASD and non-ASD students, we conducted additional ordinal regression analyses separately for the two populations. Table 5 presents the model fit results for each population.

The results in Table 5 show that both models can explain the variation in ASD students' performance in the early algebra test in a statistically significant way (Modes of representation $x^2 = 9.50, df = 4, p = 0.05$ and level of generalization $x^2 = 25.87, df = 3$, $p < 0.00$), whereas the variation in Non-ASD students' performance can be explained in a

Table 5 Model fitting information for asd and non-asd students

	Dependent variables	2 Log Likelihood	Chi-Square	df	Sig	Pseudo $R^2_{\text{Nagelkerke}}$
Mode of representations	Mode of representation ASD students	61.83	9.50	4	0.05	0.32
	Mode of representation Non-ASD students	76.71	3.86	4	0.43	0.14
Level of generalization	Level of generalization ASD students	33.97	25.87	3	0.00	0.66
	Level of generalization Non-ASD students	60.96	10.75	3	0.01	0.35

Table 6 Analysis of ASD and non-ASD students' relationship between the levels of generalization and early algebra performance

		ASD					Non-ASD				
		Hypothesis test					Hypothesis test				
	Parameter	Est.	*SE*	Wald	*df*	*p*	Est.	*SE*	Wald	*df*	*p*
Mode of representation	Concrete	1.84	1.03	3.20	1	0.07	0.47	0.35	1.89	1	0.17
	Figural	0.45	0.22	4.72	1	0.03	0.46	0.31	2.10	1	0.15
	Arithmetic	−0.12	0.29	0.16	1	0.69	0.18	0.37	0.24	1	0.63
	Algebraic	0.40	0.24	2.84	1	0.09	0.21	0.31	0.45	1	0.50
Level of generalization	Factual	0.57	0.22	4.72	1	0.03	0.21	0.31	0.45	1	0.50
	Contextual	0.40	0.24	2.84	1	0.09	0.53	0.24	4.95	1	0.03
	Symbolic	0.41	0.34	1.42	1	0.23	0.89	0.39	5.10	1	0.02

statistically significant way only by the level of generalization model ($x^2 = 10.75, df = 3$, $p = 0.01$).

Table 6 provides further information about the results of the ordinal regression within each group of students. According to the results, only the figural mode of representation (Estimate $= 0.45$, $SE = 0.22$, Wald $= 4.72$, $p = 0.03$) predicts ASD students' performance in the early algebra test, whereas none of the modes of representation predict non-ASD students' performance. In other words, non-ASD students may successfully respond to the early algebra test regardless of the modes of representation they apply, whereas ASD students are more successful in providing correct answers on the early algebra test when they successfully use figural representations.

Additionally, the results in Table 6 suggest that the factual level of generalization (Estimate $= 0.57$, $SE = 0.27$, Wald $= 4.38$, $p = 0.04$) predicts ASD students' performance in the early algebra test in a statistically significant way. In contrast, the contextual (Estimate $= 0.53$, $SE = 0.24$, Wald $= 4.95$, $p = 0.03$) and symbolic levels of generalization predict non-ASD students' performance. Simply put, factual generalization seems to enable ASD students to succeed in the early algebra test, while contextual and symbolic generalizations predict the performance of non-ASD students.

5.3 Excerpts from the interviews with one ASD and one non-ASD students

To further understand the relationship between the use of different modes of representation, levels of generalization, and students' performance on the early algebra test, interview excerpts are presented to illustrate how an ASD student and a non-ASD student approached the questions in the test.

Both students are 12 years old with no apparent delay in their mathematical age based on their scores in the TEMA-3 test. The ASD student is a female, and the non-ASD student is a male. These students were selected based on the richness of their verbalized reasoning during the interviews, which demonstrated a detailed and nuanced use of specific modes of representation and levels of generalization. This approach facilitates an in-depth exploration of the concepts of modes of representation and levels of generalization during the solving of figural pattern tasks without aiming for generalizability to a larger population.

However, it should be noted that these excerpts cannot be considered typical examples of how students with ASD and the non-ASD students interacted with the questions.

5.3.1 Example of an ASD student's response to the questions of the early algebraic thinking test

The following dialogue between the researcher (R) and the ASD student (ASD_S) indicates how the figural representations that the student creates through drawing support the identification of the spatial structure of the pattern.

> R: If you keep pushing tables together in a row, they ask you, how many people can sit if three tables are pushed together? You can write it down, or you can tell me.
> ASD_S: [Draws on a separate sheet of paper. After finishing the drawing] Eight people (see Fig. 3).
> R: How many people can sit if four tables are pushed together?
> ASD_S: [Draws 4 tables and adds the chairs, see Fig. 4] Ten.
> R: How many people can sit if five tables are pushed together?
> ASD_S: [Draws 5 tables and adds the seats, counting one by one, and writes: "12 people can sit", see Fig. 5]

Fig. 3 Response to the question involving 3 tables. [English Translation: "Eight people can sit"]

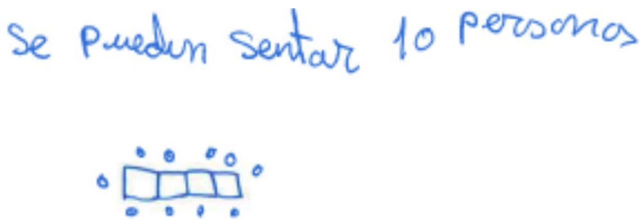

Fig. 4 Response to the question involving 4 tables. [English Translation: "10 people can sit"]

Fig. 5 Response to the question involving 5 tables. [English Translation: "12 people can sit"]

Fig. 6 Response to the question involving 8 tables. [English Translation: "18 people can sit"]

This quotation shows that the student creates a new drawing for every question, consistently counting the seats from the beginning, an approach closely tied to the drawings. This denotes a factual generalization. However, a shift in the student's perception of the pattern becomes apparent in the following question, which asks how many people can sit at eight tables.

R: Okay. Let's continue. How many people can sit if eight tables are pushed together?
ASD_S: [Without speaking, draws the eight tables with the seats around them and writes: "18 people can sit", see Fig. 6]

This quotation shows that the student still draws a representation, though it is not obvious whether she continues to count the seats one by one, as she is not counting aloud, or whether she has found a more efficient way to determine the total number of seats.

R: And if eighteen tables are pushed together?
ASD_S: [Draws 18 tables but doesn't draw seats around them. Instead, writes vertically 18+18+2, see Fig. 7] On this side, 18 people can sit [pointing with the pen to the top part of the drawing], and on this side, the same, plus two more. [Solves 18+18+2]. Thirty-eight people.
R: And if one hundred tables are pushed together?
ASD_S: [Directly responds by writing on the paper: "202 people can sit"]
R: Okay, how do you know that?
ASD_S: Well... because imagine that here [pointing to the previous drawing with 18 tables] there were 100 tables. So, 100 people could sit here, and 100 here, so 200 if we add those 100+100, and then there would be one more here and another one here. If we add it all up, 202 people.

In this dialogue excerpt, the student recognizes the overall pattern structure. The phrase "On this side…and on this side" shows the student noticed a consistent number of seats at the top and bottom of the tables, matching the number of tables. She also

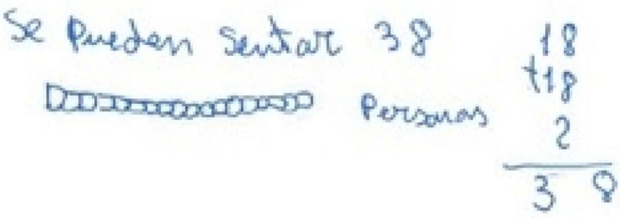

Fig. 7 Response to the question involving 18 tables. [English Translation: "38 people can sit"]

understands the addition of two extra seats, referring to those at the edges. This description seems to help her conceptualize a new operational scheme reflecting the correspondence between tables and seats, allowing her to quickly calculate the number for 100 tables. The student's focus on the arrangement of chairs and tables demonstrates contextual generalization.

Nevertheless, in the following dialogue excerpt, the student's answer implies the extraction of a generalization of the rule of the pattern, as she verbally expresses the relationship between any number of tables and the corresponding number of seats.

R: If you know the number of tables, how do you know the number of people that can sit?
ASD_S: Well, if you know the number of tables, you know that on this side there's the same amount, so then you add them up and count the ones at the ends.
R: Okay, let's see if you can write it down the way you just told me.
ASD_S: [writes: "if you know the number of tables, you know that on two of the sides there are the same, and you just have to account for the corners"]

In conclusion, this dialogue excerpt illustrates the student's evolving understanding of the relationship between tables and seats as she navigates through the questions. Initially, her method relies heavily on factual generalization through drawing and counting. However, her perception shifts toward a more structured understanding of the pattern. The recognition of a consistent relationship between the number of tables and seats, along with her awareness of the additional seats at the edges, indicates the development of a contextual generalization. This progression highlights the importance of her drawings as visual aids in fostering a deeper understanding of the pattern structure.

5.3.2 Example of a non-ASD student's response to the questions of the algebraic thinking test

The following dialogue illustrates how a non-ASD student (Non-ASD_S), much like the ASD student, develops a contextual generalization. In answering the questions involving 3, 4, and 5 tables, the student provides the correct answer without drawing and by using recursive reasoning (e.g., for three tables: "Each table adds 2 to the total"). From then on, this student employs spatial explanations for all questions from the outset and seems to notice different components of the pattern's structure.

R: How many people can be seated if 8 tables are joined?
Non-ASD_S: Well, same as always. I placed 8 tables here, and then I thought: on one edge there are 3, on the other edge there are 3, and then there are 6 tables left, 6 times 2 is 12, and that gives me 18. I add it up, and that gives me 18. Do you understand? If you want, I can draw it for you.
R: Okay.
Non-ASD_S: [He draws the figure]: Here are 1, 2, 3, 4, 5, 6, 7, and 8 tables. I imagined this, right? And I took 1 and 8 and crossed them out. That's 3. I did 3 + 3 equals 6. And then I did this: 8 − 2 is 6, and then I took 6 × 2 equals 12, and then I took 12 + 6, which gives 18 (see Fig. 8).

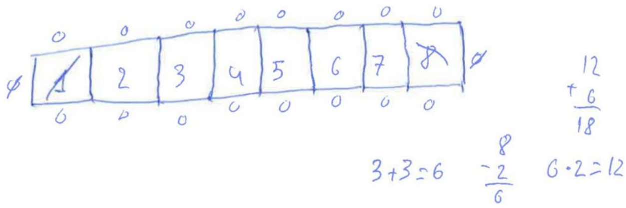

Fig. 8 Response to the question involving 8 tables

Note that the student first answered by imagining the arrangement instead of drawing and performed the operations mentally, as he explained. The student here expresses in a clear way a contextual generalization when he states: "I placed 8 tables here and then I thought: on one edge there are 3, on the other edge there are 3, and then there are 6 tables left.". While he refers to the specific case of 8 tables, he seems to have extracted a general operational scheme for finding the number of seats for every case.

In the following excerpt, when asked about 100 tables, the student explains a different method for calculating the total number of seats. The student appears to have developed an alternative operational scheme, which is also based on the spatial structure of the pattern.

> Non-ASD_S: Uh, well, no. Actually, no. It was easier because it was 100, so I didn't have to think much; the answer came to me suddenly. This time, I visualized 100 of these tables, crossed out the 2 on the corners, and then calculated 100 times 2 plus 2 equals 202.

When asked about the general rule, he expressed that he had identified two rules:

> Non-ASD_S: Oh, well, option one is to multiply that number and add 2 [writes "a) multiply that number and add 2"] umm, I mean…, multiply that number, um, yes, by 2 and… [crosses out "and add 2" and corrects it to read "multiply that number by 2 and add 2"]. Then there's another option, to subtract two… [he says while he writes: "b) subtract 2, then multiply it by two, and then add 6"] there it is. Those are the two options I see. Well, the ones I've used.

Overall, this dialogue excerpt shows that the student can analyze the pattern's spatial structure in two ways and understands that either method can be used to determine the number of seats for any number of tables. However, the generalization remains purely contextual, as the student offers spatial explanations but does not recognize that both methods lead to the same simplified symbolic expression: $2(n-2) + 6 = 2n-4 + 6 = 2n + 2$. This suggests that while the student understands the spatial elements, he has not yet developed a symbolic generalization or the ability to translate his findings into symbolic representations.

6 Discussion

The current study provides insights about similarities and differences, as well as on predictors of early algebra performance between ASD and non-ASD students. These findings highlight both anticipated and surprising outcomes that have significant implications for educational strategies.

The quantitative analyses of students'performance on the arithmetic test showed that non-ASD students outperformed ASD students, which aligns with existing research (Fernández-Cobos & Polo-Blanco, 2024; Fernández-Cobos et al., 2025), suggesting that ASD students may struggle with developing early mathematical abilities due to difficulties in understanding fundamental concepts and procedures. However, contrary to our hypothesis, there was no significant difference in the early algebra test scores between ASD and non-ASD students. This unexpected result indicates that, despite challenges in early math, ASD students may use compensatory strategies, like visual strategies through drawings, allowing them to perform similarly to their non-ASD peers in early algebra tasks.

Further statistical analyses revealed factors predicting early algebra performance, showing differences between ASD and non-ASD students. For ASD students, the figural mode of representation significantly predicted their performance, suggesting that creating visual representations is key in helping them recognize functional relationships in pattern tasks, consistent with previous research on mathematical learning (Barnet & Cleary, 2019; Goñi-Cervera et al., 2022; Polo-Blanco et al., 2024). ASD students seem to benefit from visual aids that make abstract concepts, such as functional relationships in figural patterns, more concrete and accessible. In contrast, non-ASD students' performance was not affected by representation mode, showing greater flexibility in solving pattern tasks.

The two groups also differed with respect to their levels of generalization. For ASD students, factual generalization was a significant predictor of early algebra performance. This suggests that ASD students thrive on concrete, example-based thinking, which aligns with their often detail-oriented cognitive style and reliance on specific, tangible information (Minshew et al., 2002).

For non-ASD students, both contextual and symbolic generalizations were significant predictors of early algebra performance. These findings suggest that non-ASD students can apply broader contextual insights to new situations and integrate them with abstract reasoning to tackle pattern tasks and ultimately identify correspondence relationships (Smith, 2008). This integration reflects a more sophisticated level of early algebraic thinking, enabling them to answer questions about distant terms and express the general rule of the figural pattern effectively.

The results of the qualitative analysis of students' responses to the early algebra test provide further insight into the role of specific modes of representation and levels of generalization in reaching a correct answer. The ASD student first relies on factual generalization through drawing and counting but gradually recognizes a consistent pattern and the extra seats at the edges, reflecting a shift toward contextual generalization. Her drawings serve as crucial visual aids in deepening the grasp of the pattern's structure. This student's behavior aligns with the results of the quantitative analysis, which showed that the use of drawings and factual generalization predicts ASD students' performance. These two factors appear to serve as tools for analyzing relationships in figural pattern tasks and seem to act as a foundation for progressing toward more advanced levels of algebraic thinking.

The dialogue excerpt shows that the non-ASD student answers questions about consecutive terms correctly using recursive reasoning, without needing to draw, unlike the ASD student. He appears to rely on spatial explanations when the question involves a far term aligning with the results of the quantitative analyses, which indicate that contextual generalization is a predictor of performance. While the quantitative analyses show that symbolic generalizations predict non-ASD students'performance, in this example, the student's generalization stays at a contextual level, offering spatial explanations without recognizing the symbolic expression modeling the pattern.

Overall, the study results show that although there were significant differences in the arithmetical abilities of ASD and non-ASD students, their performance in a pattern task involving functional relationships was similar, as their total scores did not significantly differ. However, it should be noted that the wide range of participants' ages (6 to 12 years), a period characterized by significant developmental variability in the acquisition of algebraic thinking, is an important factor to consider when interpreting the within-group variance observed in both the ASD and non-ASD groups.

Finally, the characteristics of early algebraic thinking, especially in figural pattern tasks focusing on structure (Blanton et al., 2015), seem to align with the cognitive style of ASD students. Therefore, educators should use visual learning tools and concrete examples in teaching methods for ASD students, fostering their engagement and understanding of algebraic relationships. This approach could lay the groundwork for their progression toward using alphanumeric symbols and higher levels of generalization.

7 Conclusions

Despite the valuable insights gained from this study on early algebraic thinking among students with ASD and their non-ASD peers, several limitations must be acknowledged. First, the small sample size limits the generalizability of the results. Second, the study focused on a specific algebraic content strand and pattern task, which may not capture the full range of algebraic thinking abilities. Third, prior mathematical knowledge or experiences with similar tasks were not controlled due to the large number of schools in which they were enrolled, a consequence of the challenges associated with recruiting ASD participants. Fourth, while the study uses autism to explain the results, other factors may also influence students' behavior, and cognitive variability within both ASD and non-ASD groups was not examined. Finally, the study does not consider arithmetic abilities as a potential predictor of performance, which could be addressed in future research.

In conclusion, this study shows that while students with and without ASD achieve similar scores on early algebra pattern tasks, they use different processing strategies. The findings highlight the importance of recognizing and addressing these differences to improve outcomes for all students. Specifically, the study emphasizes the value of drawings in helping ASD students engage with algebraic tasks. Visual representations, like drawings, provide crucial support in understanding abstract concepts, particularly functional relationships. Algebra education should integrate visual aids and drawing-based strategies, considering these tools' value. Tailored instructional approaches that leverage students'cognitive strengths can foster more inclusive learning environments. Additionally, the study suggests that ASD students should not be assessed solely on broad traits but through more focused research on their specific strengths and challenges in mathematics. Future research should explore how to encourage the drawing-based strategies of ASD students and use these to help them develop symbolic representations and generalizations, areas where they may struggle compared to non-ASD peers.

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Data availability The data that support the findings of this study are available in the supplementary material.

Declarations

Conflicting interests The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Editorial board members and editors We declare that Demetra Pitta-Pantazi, author of this study, is one of the Associate Editors of the journal.

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