



Cognitive Predictors of Early Algebra Performance in Students With and Without Autism

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Abstract

Purpose Students with autism spectrum disorder (ASD) are increasingly included in mainstream classrooms; however, little is known about how they approach early algebra tasks. This study investigates how different cognitive abilities support early algebraic thinking in students with and without ASD, focusing on performance in a figural pattern generalization task.

Methods The sample comprised 52 children aged 6 to 12 years (26 with ASD and 26 without ASD), all without intellectual disabilities. Participants were assessed on working memory, processing speed, spatial reasoning, verbal reasoning, fluid reasoning, theory of mind, and mathematical age, which reflects early arithmetical ability relative to age-based expectations. Regression analyses were conducted to identify cognitive predictors of early algebra performance in each group.

Results For students without ASD, fluid reasoning and mathematical age were significant predictors of performance. In contrast, for students with ASD, spatial reasoning and theory of mind emerged as significant predictors. These findings indicate distinct cognitive pathways underlying early algebraic thinking across groups.

Conclusion The results demonstrate that cognitive predictors of early algebra differ meaningfully between students with and without ASD. These findings have important implications for inclusive education, highlighting the need to tailor instructional approaches to diverse cognitive profiles.

Keywords Early algebraic thinking · Autism spectrum disorder (ASD) · Cognitive abilities · Pattern generalization

Introduction

Autism spectrum disorder (ASD) is one of the most prevalent neurodevelopmental conditions encountered in school-aged children (CDC, 2025). As educational systems strive toward greater inclusivity, an increasing number of students with ASD are participating in mainstream classrooms in both the United States (Granieri et al., 2025) and across Europe (van Kessel et al., 2021). This shift presents both opportunities and challenges, particularly in understanding and addressing the diverse learning needs of diverse students. While inclusive policies promote access, meaningful participation requires tailored instructional strategies grounded in a robust understanding of how students with

ASD think and learn (Olsson & Nilholm, 2022). Despite growing research on the cognitive profiles of children with ASD, many aspects of their thinking and learning in core academic domains—especially mathematics—remain underexplored (Tonizzi & Usai, 2023).

Mathematics is a fundamental part of K to 12 education, and algebra, in particular, has emerged as a central focus in mathematics curricula worldwide from the early grades (Kieran, 2018). Research has consistently supported that introducing aspects of algebraic thinking early, such as noticing and expressing generalizations of mathematical structures, supports a smoother transition to formal algebra in later grades (Ellis & Özgür, 2024; Kieran, 2022). This early exposure aligns with evolving conceptions of school algebra that extend beyond symbolic manipulation to include rich mathematical reasoning (Radford, 2018). As such, fostering algebraic thinking from a young age is now considered essential for all learners (Freiman & Fellus, 2021). However, ensuring that these opportunities are accessible and effective for neurodiverse students requires

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understanding how different learners engage with core algebraic concepts and procedures.

Contrary to popular assumptions that individuals with ASD excel in mathematics, empirical findings suggest a more complex picture. For example, approximately 20% of students with ASD exhibit signs of a mathematics learning disability—far higher than the 7% observed in the general population, while only 4% of students with ASD demonstrate mathematical giftedness (Geary, 2011; Oswald et al., 2016). Some of these challenges may stem from differences in domain-general cognitive abilities. Research indicates that students with ASD often experience difficulties in working memory (Bennetto et al., 1996; Wang et al., 2017), inhibitory control (Geurts et al., 2014; Tonizzi et al., 2022), and processing speed, while also demonstrating strengths in areas such as visuospatial processing (Baron-Cohen, 2002; Shah & Frith, 1993). Other cognitive abilities, like fluid reasoning, show more variability across individuals and tasks (Assouline et al., 2012; Chen et al., 2019). These cognitive characteristics are likely to influence mathematical learning, yet most existing studies on ASD and math focus on basic arithmetic or problem-solving performance (e.g., Polo-Blanco et al., 2024; Tonizzi & Usai, 2023), with little attention paid to algebra.

On the contrary, the relationship between cognitive abilities and algebraic thinking has been increasingly studied in typically developing students (TD). For instance, working memory, inhibition, and cognitive flexibility have been linked to performance on algebraic tasks such as solving equations and generalizing patterns (Bull and Scerif 2001; Chimoni et al. 2023a; Swanson and Beebe-Frankenberger 2004). Furthermore, research has shown that different forms of domain-general cognitive abilities related to reasoning—such as analogical, inductive, spatial, and deductive reasoning—predict student performance in early algebra, with their influence varying across age groups from 9 to 12 years old (Chimoni et al. 2023b). These findings support the idea that early algebra engages a unique set of cognitive abilities.

However, to date, no studies have directly examined how the distinct cognitive profiles of students with ASD relate to their algebraic thinking abilities. This represents a significant gap in the literature. While we know that typically developing students rely on a range of cognitive abilities to succeed in early algebra, it remains unclear whether students with ASD draw on the same cognitive resources or whether different patterns of strengths and difficulties shape their engagement with algebraic tasks. Emerging evidence from studies focused on arithmetic and algebra suggests that the cognitive factors influencing math performance in ASD students may differ in both nature and magnitude from those seen in their non-autistic peers (e.g., Fernández-Cobos et al., under review; Polo-Blanco et al., 2024, 2025,

in press). This raises critical questions about whether the models developed with typically developing students can adequately capture the models of neurodiverse learners in algebra.

Given the central role of algebra in mathematical development and the growing presence of students with ASD in inclusive classrooms, it is essential to investigate how different cognitive abilities predict algebraic thinking in young learners with ASD. Moreover, comparing these relationships with those observed in non-ASD students can provide valuable insights into whether shared or distinct cognitive mechanisms support algebraic thinking across groups. Addressing this gap will not only advance theoretical understanding of algebra learning in neurodiverse populations but also inform the design of cognitively informed instructional practices in early algebra. The current study aims to fill this gap by investigating how various cognitive abilities may predict early algebra performance in both ASD and non-ASD students, with a specific focus on pattern generalization as a key component of early algebraic thinking.

Literature Review

Cognitive Abilities Underlying Mathematical Performance

The relationship between cognitive abilities and mathematical performance has been examined extensively in TD students, spanning arithmetic (e.g., Andersson, 2008; Berg, 2008), problem solving (e.g., Blair & Razza, 2007; Swanson & Beebe-Frankenberger, 2004), and, more recently, algebraic thinking (e.g., Chimoni et al. 2023a, b; Chimoni & Pitta-Pantazi, 2017). Among TD learners, domain-general executive functions, particularly working memory, processing speed, and cognitive flexibility, consistently scaffold success across mathematics (Andersson, 2008; Berg, 2008; Bull & Scerif, 2001; Swanson & Beebe-Frankenberger, 2004). As tasks progress from concrete computation to the structural thinking required in early algebra, additional resources become salient: fluid reasoning supports abstraction and relational mapping; verbal reasoning underpins the linguistic manipulation of symbolic statements; spatial reasoning enables mental transformation of visual-symbolic patterns; and theory of mind (ToM) may help students interpret functional relations that require perspective-taking or mapping between representational spaces (Green et al., 2017; Lockl et al., 2017; Yang et al., 2019).

Cognitive profiles within ASD often diverge from TD norms. Research documents relative strengths in rule-based reasoning and pattern detection, but weaknesses in executive-function and social-cognitive processes (Assouline

et al., 2012; Bae et al., 2015; Ozonoff & Schetter, 2007). Although most ASD mathematics studies center on arithmetic and word-problem solving, the constellation of predictive abilities appears to shift: visuo-spatial and linguistic skills jointly predict arithmetic in children with ASD (Fernández-Cobos et al., in press), whereas ToM deficits relate uniquely to problem-solving difficulties in this group (Polo-Blanco et al., 2024). To date, however, no research has explored whether and how these abilities predict algebraic thinking in ASD.

This gap raises a pivotal research question about whether the same cognitive mechanisms support algebraic abilities in students with and without ASD, or whether ASD entails a distinct constellation of supports and constraints. Addressing this question can (a) refine cognitive models of early algebra, (b) differentiate domain-general from ASD-specific predictors, and (c) inform targeted instructional design.

The remainder of this section reviews cognitive abilities that previous research has identified as key to mathematical performance—working memory, processing speed, fluid reasoning, verbal reasoning, spatial reasoning, and theory of mind (ToM)—in studies involving both TD and ASD students. We then introduce early algebraic thinking, focusing on pattern generalisation, the core algebraic activity examined in the present study. Among the various algebraic thinking abilities, the generalization of patterns is widely recognized as foundational (Kaput, 2008; Kieran, 2022) and is often introduced as early as the first grades of primary school. In fact, most researchers consider pattern generalization a key developmental route into formal algebraic thinking (e.g., Blanton et al., 2015). For this reason, it was selected as the focus of the current study. By examining the cognitive underpinnings of pattern generalization in both ASD and non-ASD populations, this study aims to shed light on shared and unique predictors of early algebra competence.

Cognitive Abilities Underlying Mathematical Performance

Working Memory

Working memory, the executive function responsible for the temporary storage and manipulation of information, is critical for mathematical performance. It supports complex tasks such as comprehension and reasoning (Baddeley, 2002). In TD students, working memory has been shown to predict performance in both arithmetic and early algebra tasks (Chimoni et al., 2023; Espy et al., 2004).

In students with ASD, challenges in working memory are common (Benetto et al., 1996; Habib et al., 2019; Wang et al., 2017) and play a significant role in explaining variability

in their mathematical performance (Assouline et al., 2012; Bullen et al., 2022; Chen et al., 2019; Marinopoulou et al., 2024; Polo-Blanco et al., 2024). In mathematics, these difficulties are particularly evident in tasks that require holding and coordinating multiple pieces of information, such as multi-step word problem solving, carrying out sequential calculations, or integrating verbal and numerical information. Previous studies found that working memory predicts math achievement in ASD, particularly in tasks involving multi-step reasoning and verbal processing (Assouline et al., 2012; Chen et al., 2019; Polo-Blanco et al., 2024). These findings reinforce that working memory is a core cognitive factor influencing the mathematical performance of ASD students.

Processing Speed

Processing speed—the ability to quickly process and respond to information—has been linked to early mathematical achievement, particularly in arithmetic, among TD students (Mashburn et al., 2024). Studies show that faster processing speed supports basic calculation and problem-solving skills in young children (Bull & Johnston, 1997; Swanson & Beebe-Frankenberger, 2004), though its influence appears to decline with age and task complexity (Berg, 2008; Floyd et al., 2003). In contrast, algebraic thinking in TD children seems to rely more on reasoning-based processes than on processing speed (Chimoni et al., 2023; Chimoni & Pitta-Pantazi, 2017).

In students with autism, slower processing speed is a common characteristic, documented across age groups and IQ levels (Assouline et al., 2012; Travers et al., 2014). In mathematics, this is reflected in difficulties with the rapid completion of basic arithmetic tasks and reduced efficiency when solving calculation-based problems. Processing speed remains a significant predictor of mathematical performance in this population, as it, together with working memory, has been shown to contribute to performance in basic arithmetic tasks (Assouline et al., 2012; Marinopoulou et al., 2024).

Fluid Reasoning

Fluid reasoning—the ability to solve novel problems using logic and inference—has been widely associated with mathematical development in TD students. Longitudinal and cross-sectional studies have consistently shown that fluid reasoning predicts both calculation skills and mathematical reasoning across ages (Floyd et al., 2003; Green et al., 2017; Taub et al., 2008). Among various cognitive predictors, Green et al. (2017) found fluid reasoning to be the most robust factor explaining math achievement over time.

On the contrary, studies involving autistic students have not found a consistent link between fluid reasoning and mathematical performance (e.g., Bullen et al., 2020; Fernández-Cobos et al., under review; Polo-Blanco et al., 2024). Despite its recognized importance in TD populations, fluid reasoning does not appear to play a central role in explaining individual differences in math skills among children with ASD, at least within the domains and age ranges studied so far.

Visuospatial Skills

Visuospatial skills involve understanding spatial relationships, mental manipulation of objects, and interpreting visual information (Fung et al., 2020). In TD children, these skills are linked to numerical representation (e.g., mental number line), problem solving, and spatial reasoning in mathematics (Gunderson et al., 2012). Studies suggest that visuospatial abilities may predict mathematics achievement more strongly than some executive functions during early development (Fung et al., 2020).

In ASD students, visuospatial ability is often considered a cognitive strength, with individuals showing superior visual search and visual thinking (Baron-Cohen, 2002; Karmakar & Sarkar, 2020). Research indicated that visuospatial skills support early numeracy and problem solving in ASD, possibly compensating for verbal challenges (Fernández-Cobos et al., in press; St. John et al., 2018). Additionally, the frequent use of drawings and visual elements by ASD students during their engagement with algebra tasks (Bruno et al., 2024; Goñi-Cervera et al., 2022; Polo-Blanco et al., 2025) reflects a visually oriented thinking style that may help compensate for verbal communication challenges. However, Kim and Cameron (2016) caution that visuospatial strengths alone may not translate into improved performance on more complex mathematical tasks, which also rely heavily on executive functions like planning, cognitive flexibility, and working memory—areas that are often challenging for students with ASD.

Verbal Reasoning

Verbal reasoning refers to the ability to understand, analyze, and reason using concepts framed in words. It involves skills like comprehension, inference, and the ability to manipulate language to solve problems or make decisions (Weschler, 2015).

The role of language in mathematics varies with the type of mathematics skill assessed. In TD students, language strongly supports early arithmetic and problem solving, especially tasks involving word problems. For example, phonological processing and expressive grammar predict

preschoolers' math development (Viesel-Nordmeyer et al., 2022), while domain-specific mathematical language is a stronger predictor of numeracy than general vocabulary (Purpura & Reid, 2016). Meta-analyses confirm that language is more closely linked to mathematical tasks with high linguistic demands, such as word problems and comprehensive math skills (Peng et al., 2020).

In ASD students, language difficulties are common and can influence math performance (e.g., Bae et al., 2015). Fernández Cobos et al. (in press) found that in young ASD children without intellectual disability, a combination of visuospatial and language skills best predicted early mathematical performance, with those struggling in math showing lower language abilities. In the context of arithmetic problem solving, Polo-Blanco et al. (2024) reported that verbal comprehension deficits were associated with poorer word problem solving in older ASD students, alongside challenges in inhibition and theory of mind.

Theory of Mind

Theory of Mind (ToM)—the ability to understand others' mental states—has been proposed as a cognitive factor contributing to early numeracy in TD children (Lockl et al., 2017). ToM develops progressively, from recognizing that others can hold false beliefs (first-order) to understanding nested beliefs (second-order) (Miller, 2009). In autism research, numerous studies have documented consistent delays or deficits in ToM among children and adults with ASD (Baron-Cohen et al., 1985; Frith, 1994; Schuwert & Sodian, 2023), even among those with typical IQ.

Recent evidence suggests that Theory of Mind (ToM) may also relate to mathematical performance in ASD. Word problem solving often requires understanding implicit information, interpreting intentions embedded in the text, and integrating contextual and quantitative cues. In this line, Polo-Blanco et al. (2024) found that poorer-performing autistic children without intellectual disability showed lower scores in ToM, inhibition, and cognitive flexibility in relation to success in solving arithmetic word problems. These results suggest that ToM may support higher-level mathematical problem solving in ASD, likely in interaction with other executive and linguistic skills.

Early Algebra and Pattern Generalization in TD and ASD

Despite strong evidence that cognitive abilities influence early algebra performance in TD students, little is known about how these abilities predict early algebraic thinking in autistic students. This study addresses that gap by focusing on one core early algebra concept: pattern generalization.

Among the various strands of algebraic thinking, the ability to generalize patterns is widely recognized as foundational (Kaput, 2008; Kieran, 2022).

Pattern generalization involves identifying regularities across cases and expressing the underlying structure of a relationship (Blanton et al., 2015; Radford, 2008). This can take many forms: numerical, verbal, visual, or symbolic. In figural pattern tasks, students analyze how quantities change and relate to each other, such as expressing the rule $y = 4x + 1$ from a visual sequence. These tasks support functional thinking, a key entry point into algebra that shifts students' attention from isolated values to invariant relationships between variables (Rivera, 2010; Warren & Cooper, 2008).

Students typically progress through levels of generalization, from factual (e.g., “add 4 each time”), to contextual (e.g., verbal descriptions of structure), to symbolic (e.g., algebraic rules) (Cooper & Warren, 2011; Radford, 2003). These levels also reflect varied modes of representation, such as gestures, diagrams, language, or symbols (Radford, 2018). Crucially, students may reach the same correct answer using different strategies and representations, highlighting the diversity in how generalizations are formed and expressed (Blanton et al., 2015).

Instruction and visual supports can enhance students' ability to generalize, especially by helping them connect spatial and numerical information (Goñi-Cervera et al., 2024; Mason et al., 2009). While most prior work has focused on TD learners, Polo-Blanco et al. (2025) found no significant differences in pattern generalization performance between autistic and non-autistic students. This raises important questions about whether similar outcomes may arise from the same or different underlying cognitive processes in the ASD group. Understanding how cognitive abilities contribute to generalization in autism is therefore essential for developing targeted and inclusive instructional strategies.

Aim of the Study and Research Question

This study aims to identify the cognitive abilities that predict early algebra performance in pattern generalization in students with and without ASD. Additionally, students' arithmetic abilities were considered, given prior research suggesting differences between TD and ASD students that may influence this relationship. The cognitive abilities examined include working memory, processing speed, verbal reasoning, spatial reasoning, fluid intelligence, and theory of mind. The research question is as follows:

Which cognitive abilities (working memory, processing speed, verbal reasoning, spatial reasoning, fluid intelligence, and theory of mind), along with basic arithmetic

skills, are the most significant predictors of early algebraic thinking in students with and without ASD?

Methods

Study Design

The study employed an exploratory design due to limited existing knowledge about the extent to which various cognitive abilities and basic arithmetical abilities predict early algebraic thinking abilities in students with and without ASD, and whether these predictive relationships differ between the two groups. This approach allows for the identification of unique or shared predictors within each group.

Participants

The study included 26 participants in the ASD group (23 males and 3 females; mean age=9.35 years) and 26 participants in the non-ASD group (also 23 males and 3 females; mean age=9.41 years). All participants ranged in age from 6 to 12 years and did not have intellectual disabilities ($IQ \geq 70$, assessed using the WISC-V; Wechsler, 2015).

Participants in the ASD group were recruited from various healthcare, educational, and social services supporting individuals with autism in a Spanish region. These included child psychiatry and pediatric outpatient clinics, family associations, and school counseling services. Recruitment occurred between July 2019 and February 2021. ASD diagnoses had been previously established through specialized health services and were subsequently confirmed by a psychiatrist from the research team through review of clinical records and direct clinical interviews with participants and their families, according to DSM-5 criteria (APA, 2013).

Inclusion criteria for the ASD group were: (1) a confirmed ASD diagnosis and absence of psychiatric or neurodevelopmental comorbidity, including ADHD, dyslexia, and language disorders; (2) a Full-Scale IQ (FSIQ) ≥ 70 , assessed using the Wechsler Intelligence Scale for Children—Fifth Edition (WISC-V; Wechsler, 2015); (3) age between 6 and 12 years; and (4) a minimum raw score of 26 on the Test of Early Mathematics Ability (TEMA-3; Ginsburg & Baroody, 2007). This score corresponds to an equivalent mathematical age of approximately 5 years and 6 months and ensured a minimum level of basic arithmetical knowledge required for the tasks included in the study. The absence of comorbid conditions was verified through both clinical record review and the psychiatric interview conducted by the research team. For each ASD participant, a matched non-ASD peer—of the same sex, age, school, grade, and classroom—was identified through school counselors or administrators.

No direct information was collected regarding the specific algebraic concepts taught to participants at each school, due to logistical challenges associated with recruiting across multiple institutions. Nevertheless, the Spanish national curriculum in effect at the time (Ministerio de Educación, Cultura y Deporte, 2014) introduces algebraic thinking as early as first grade, emphasizing that students should “recognise and describe regularities and patterns in numerical, geometric, and functional contexts.” This suggests that participants were likely exposed to relevant instructional content supporting early algebraic thinking.

Parents or legal guardians were provided with a detailed explanation of the study’s aims and procedures before providing written informed consent. This study was previously approved by the Cantabria Research Ethics Committee).

Data Collection Procedure

This study forms part of a broader research project aimed at examining the mathematical abilities of ASD students compared to non-ASD students, as well as their connection to various cognitive abilities. Each participant took part in two to three sessions conducted by a psychology team to assess cognitive abilities. Additionally, another two to three sessions were led by mathematics education researchers to evaluate students’ mathematical performance. The present paper focuses specifically on the sessions related to early algebraic thinking. All sessions were videotaped and transcribed for subsequent analysis.

Instruments

WISC-V

The WISC-V (Wechsler Intelligence Scale for Children – Fifth Edition) is a widely used standardized test designed to assess the intellectual abilities of children aged 6 to 16 years. It measures cognitive abilities across five primary domains: Verbal Comprehension, Spatial Reasoning, Fluid Reasoning, Working Memory, and Processing Speed. The test provides a Full-Scale IQ (FSIQ) score as well as individual index scores, offering a comprehensive profile of a child’s cognitive strengths and weaknesses. Full-Scale IQ (FSIQ) was used as an inclusion criterion ($FSIQ \geq 70$). As the study focused on specific cognitive domains, FSIQ was not included as a separate variable in the analyses to avoid overlap with the more fine-grained cognitive measures.

Within the WISC-V, the Working Memory Index assesses the ability to hold and mentally manipulate information over short periods, which is crucial for learning and following instructions. The Processing Speed Index assesses how quickly and accurately a child can complete simple

cognitive tasks, often under time pressure, reflecting efficiency in basic mental operations. The Verbal Comprehension Index assesses the ability to understand, use, and think with spoken language, reflecting skills such as vocabulary knowledge and verbal reasoning. The Spatial Reasoning Index assesses the ability to analyze and manipulate visual information, including understanding spatial relationships and constructing designs. Finally, the Fluid Reasoning Index assesses a child’s capacity for logical thinking and problem-solving with novel or abstract information, such as identifying patterns.

Theory of Mind Test

The NEPSY-II (A Developmental NEuroPSYchological Assessment, Second Edition) includes subtests that assess various social cognitive functions in children aged 3 to 16 years, including Theory of Mind—the ability to understand that others have thoughts, beliefs, intentions, and perspectives different from one’s own. The Theory of Mind subtest specifically evaluates a child’s understanding of social cognition through tasks that involve recognizing emotions, interpreting others’ beliefs, and predicting behavior based on mental states. This component is particularly valuable for identifying social-cognitive difficulties often seen in neurodevelopmental conditions such as ASD.

Mathematical Age TEMA-3 Test

The TEMA-3 (Test of Early Mathematics Ability, Third Edition) is a standardized assessment designed to measure the basic arithmetical abilities of children aged 3 to 8 years. It evaluates both formal (e.g., numeral recognition, calculation, number facts) and informal (e.g., number sense, counting, comparison) arithmetical abilities. The test provides a Mathematics Age score, which indicates the level of mathematical development compared to typical age norms, allowing educators and researchers to assess whether a child’s arithmetical abilities are above, below, or at the expected developmental level.

Early Algebra Test

To evaluate students’ early algebraic abilities, a written test was developed that included a figural pattern task adapted from Carraher et al. (2008), based on the linear function $f(x) = 2x + 2$. This type of task has been widely used in research with elementary school students (e.g., Blanton et al., 2015) and was considered appropriate for the participants in this study, as the Spanish mathematics curriculum in force at the time (Ministerio de Educación, Cultura y Deporte, 2014) places emphasis on the development of

algebraic thinking from Grade 1, highlighting abilities such as figural pattern recognition and generalization.

The task was administered during individual semi-structured interviews. It began with a scenario involving the arrangement of one and two tables and the number of people seated around them (see Fig. 1). This was followed by seven questions targeting both specific near and far terms (e.g., “If there are 3/4/5/8/18/100 tables, how many people can sit around them?”) and a general rule (e.g., “If you know the number of tables, how can you determine how many people can sit around them?”). The interviewer provided reading assistance when necessary and encouraged students to explain their reasoning for each response. Each correct answer was awarded one point, with a maximum score of 7. The Cronbach’s alpha was 0.853, indicating a high level of internal consistency and suggesting that the task reliably measured early algebraic thinking abilities.

Data Analysis

To gain an overview of students’ performance, descriptive statistics were conducted using SPSS, along with a one-way ANOVA to examine statistically significant differences between the two student groups. Subsequently, multiple linear regression analyses were conducted separately for students with ASD and those without ASD. In each analysis, the dependent variable was the students’ score on the early algebra test. The independent variables included measures of the cognitive abilities (working memory, processing speed, spatial reasoning, verbal reasoning, fluid intelligence, theory of mind) as well as mathematical age. Mathematical age was included to control for individual differences in basic arithmetical abilities, enabling a clearer examination of the unique contributions of cognitive abilities and theory of mind on early algebraic thinking abilities.

Considering that the age of the participants ranged from 6 to 12 years, we assessed whether heteroscedasticity was present in the data, specifically, whether the variability of early algebra scores was different for younger students compared to older students, with scores becoming unpredictable or scattered as students got older. To do this, we performed Levene’s test for homogeneity of variances, grouping age into discrete categories (e.g., 6–7, 8–9, 10–11, 12–13 years). The results indicated a statistically significant p -value ($p < .001$), leading us to reject the null hypothesis of equal variances. Therefore, the results confirmed that the early algebra scores did indeed vary much more in some age groups than others. This provided clear evidence of heteroscedasticity, thereby supporting the decision to adjust for unequal variance across age groups in our regression analysis.

In response, we employed Weighted Least Squares (WLS) regression, using weights inversely proportional to the variance of early algebra scores within each age group. In this way, more weight was given to the age groups with more consistent scores and less weight to the highly unpredictable groups. Using WLS enhanced the precision of the regression estimates and ensured that the model more accurately reflected the underlying structure of the data.

In conducting the multiple linear regression analysis, several key indices and assumptions were assessed to ensure the validity of the results. First, model fit was evaluated through the R^2 value, which indicates the proportion of variance in the dependent variable explained by the independent variables, that is how well our overall model explained the changes in students’ early algebra scores. We also examined the adjusted R^2 to account for the number of predictors used. Second, to examine the significance of individual predictors, we checked standardized beta coefficients, t -values, and associated p -values (with significance set at $p < .05$). In this way, the contribution of each individual predictor to the model was assessed. Finally, multicollinearity among

Fig. 1 The pattern generalization task

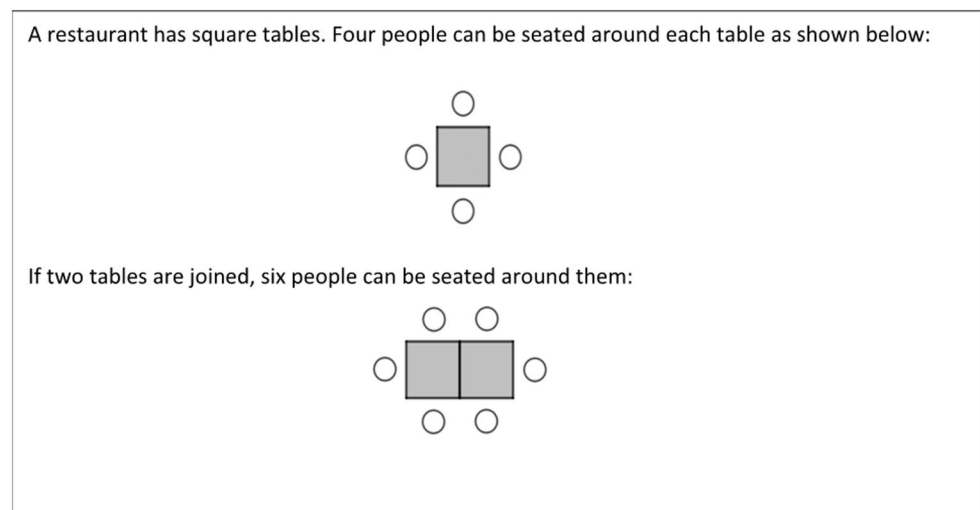


Table 1 Descriptive statistics and one-way ANOVA results for ASD and non-ASD student groups

Measure	ASD students		Non-ASD students		F	p
	M	SD	M	SD		
Working memory	0.56	0.08	0.63	0.07	10.20	.002*
Processing speed	0.52	0.09	0.63	0.09	22.26	<.001*
Spatial reasoning	0.60	0.08	0.62	0.08	1.15	.289
Verbal reasoning	0.56	0.12	0.66	0.07	11.65	.001*
Fluid Reasoning	0.63	0.08	0.65	0.09	0.31	.582
Theory of mind	0.63	0.26	0.86	0.02	19.26	<.001*
Mathematical age	0.63	0.09	0.70	0.04	8.34	.006*
Early algebraic thinking	0.31	0.35	0.42	0.32	1.38	.246

Significant differences are indicated where $p < .05$

Table 2 Results from the multiple linear regression for the non-ASD group with early algebraic thinking abilities as the dependent variable

Predictor	β	t	p
(Constant)		-2.223	.037
Working Memory	-.040	-0.021	.827
Processing Speed	.251	1.521	.143
Spatial Reasoning	-.129	-0.610	.548
Verbal Reasoning	.086	0.488	.631
Fluid Reasoning	.508	3.110	.005*
Theory of Mind	.207	1.066	.299
Mathematical Age	.397	2.430	.024*

β =standardized regression coefficient

The overall model was statistically significant, $F(7, 22)=7.74$, $p=.003$, $R^2=.413$, adjusted $R^2=.360$, $f^2=0.70$

Significant differences are indicated where $*p < .05$

predictors was evaluated using Variance Inflation Factor (VIF) values, with VIFs below 5 indicating acceptable levels. This test was used to ensure that the individual predictors weren't overlapping or repeating the same information.

Results

Descriptive Statistics and Differences Between Students With and Without ASD Across Cognitive Abilities, Mathematical Age, and Early Algebraic Thinking

Table 1 presents the means and standard deviations for both ASD and non-ASD student groups across various cognitive and mathematical measures. Overall, students without ASD showed significantly higher performance in most measures of cognitive abilities and mathematical age, while both groups were found to perform similarly only in spatial reasoning, fluid reasoning, and the early algebra test. One-way ANOVA results revealed significant group differences in mathematical age ($F=8.34$, $p=.006$), working memory, ($F=10.20$, $p=.002$), processing speed ($F=22.26$, $p<.001$), verbal reasoning ($F=11.65$, $p=.001$), and theory of mind ($F=19.26$, $p<.001$). These results indicate that students

without ASD significantly outperformed those with ASD in these areas. In contrast, no statistically significant group differences were observed in spatial reasoning ($F=1.15$, $p=.289$), fluid reasoning ($F=0.31$, $p=.582$), or early algebraic thinking ($F=1.38$, $p=.246$). This suggests that students across both groups exhibited comparable performance in these domains. The presence of both shared and distinct patterns of performance justifies the use of separate regression models for each group.

The Regression Model for the Non-ASD Students

Table 2 presents the results of the multiple linear regression analysis for the non-ASD student group. The assumption of multicollinearity was met, as all VIF values were below 5 and close to 1, indicating that multicollinearity was not a concern and that the predictors contributed independently to the model.

The model was statistically significant ($F=7.74$, $p=.003$) and accounted for 41.3% of the variance in early algebra scores ($R^2=0.413$, adjusted $R^2=0.360$). This represents a large overall effect size ($f^2=0.70$), indicating that the included cognitive predictors have a meaningful impact on early algebraic performance. Among the predictors, fluid reasoning ($\beta=0.508$, $t=3.110$, $p=.005$) and mathematical age ($\beta=0.397$, $t=2.430$, $p=.024$) emerged as statistically significant positive predictors. In contrast, working memory, processing speed, spatial reasoning, verbal reasoning, and theory of mind did not significantly contribute to the model ($p > .05$ for all). These findings suggest that for students without ASD, early algebraic thinking is primarily supported by fluid reasoning and foundational arithmetical skills.

The Regression Model for the ASD Students

Table 3 presents the results of the multiple linear regression analysis for the group of the ASD students. As with the model for non-ASD students, all VIF values in the ASD student model were below 5 and close to 1, indicating that

Table 3 Results from the multiple linear regression for the ASD group with early algebraic thinking abilities as the dependent variable

Predictor	β	t	p
(Constant)		-2.918	.008
Working Memory	-.035	-0.256	.801
Processing Speed	.140	0.986	.336
Spatial Reasoning	.333	2.498	.021*
Verbal Reasoning	.161	1.176	.254
Fluid Reasoning	.052	0.357	.725
Theory of Mind	.681	5.116	<.001*
Mathematical Age	-.169	-0.852	.404

β =standardized regression coefficient

The overall model was statistically significant, $F(7, 22)=18.19$, $p < .001$, $R^2 = .634$, adjusted $R^2 = .599$, $f^2 = 1.73$

Significant differences are indicated where * $p < .05$

multicollinearity was not a concern and that the predictors contributed independently to the model.

The regression model for ASD students was statistically significant ($F = 18.19$, $p = .000$) and accounted for 63.4% of the variance in early algebra scores ($R^2 = 0.634$, adjusted $R^2 = 0.599$). This represents a very large overall effect size ($f^2 = 1.73$), indicating that the cognitive predictors under examination have a dominant role in explaining early algebra performance among students with ASD. Among the predictors, theory of mind ($\beta = 0.681$, $t = 5.116$, $p < .001$) and spatial reasoning ($\beta = 0.333$, $t = 2.498$, $p = .021$) emerged as statistically significant positive predictors. In contrast, working memory, processing speed, verbal reasoning, fluid reasoning, and mathematical age did not significantly contribute to the model significant ($p > .05$ for all). These findings suggest that for students with ASD, algebraic thinking is primarily supported by theory of mind which reflects social-cognitive abilities and spatial reasoning, rather than by more traditional domain-general cognitive abilities or foundational arithmetical abilities.

Summary of Results

In summary, the findings provide preliminary insight into the profiles of performance and predictive mechanisms for early algebraic thinking between students with and without ASD. Comparative analyses (ANOVA) demonstrated that students without ASD outperformed their peers with ASD in foundational arithmetical abilities and most domain-general cognitive abilities (working memory, processing speed, verbal reasoning, and theory of mind). However, both groups exhibited comparable performance in spatial reasoning, fluid reasoning, and early algebraic thinking.

Despite achieving similar scores on the early algebra test, multiple linear regression models suggested that the groups had diverging cognitive abilities supporting early algebraic thinking. For students without ASD, early algebra

performance was primarily predicted by fluid reasoning and mathematical age. In contrast, for students with ASD, early algebra performance was strongly predicted by theory of mind and spatial reasoning.

However, these regression results must be interpreted with caution due to important methodological constraints. Because this study focused on a specialized population, it was not feasible to recruit a large sample or to further restrict the age criteria. Consequently, the analyses are limited by a small sample size ($N = 26$ per group) and a broad developmental span spanning ages 6 to 12. Because running multiple linear regressions with seven predictors on a sample of 26 participants limits statistical power, these models cannot be considered definitive. Instead, these findings should be treated strictly as exploratory, offering tentative pathways and hypotheses for future, larger-scale validation studies.

Discussion

This study contributes novel insights into the cognitive foundations of early algebraic thinking in both students with and without ASD. By investigating a range of cognitive abilities—working memory, processing speed, fluid reasoning, verbal reasoning, spatial reasoning, and theory of mind (ToM)—as well as basic arithmetical abilities, the study offers a comparative lens on how early algebraic thinking is differentially supported across neurodiverse learners. Specifically, the use of a figural pattern generalization task allowed for the examination of early algebraic thinking in a developmentally appropriate, non-symbolic format that emphasizes structure recognition and general rule inference (Kieran, 2022; Radford, 2018).

A key contribution of this study is identifying distinct performance profiles in the cognitive and mathematical domains between students with and without ASD, while also highlighting an area of parity. Specifically, the comparative findings of the current study demonstrate that students without ASD exhibited significantly stronger mathematical age, working memory, processing speed, verbal reasoning, and theory of mind. These results align with existing literature demonstrating that domain-general cognitive abilities, such as working memory, processing speed, and verbal reasoning, are common areas of difficulty for students with ASD, leading to standard performance gaps compared to their typically developing peers (Assouline et al., 2012; Bae et al., 2015; Travers et al., 2014). However, their performance in spatial reasoning, fluid reasoning, and critically early algebraic thinking was comparable to ASD students. These results align with documented findings that individuals with ASD often exhibit cognitive strengths in rule-based reasoning, visual search, and pattern detection

(Baron-Cohen, 2002; Ozonoff & Schetter, 2007). Overall, this finding is particularly noteworthy, as it suggests that students with ASD possess intact abilities in early algebraic thinking tasks that match those of typically developing peers, thereby establishing a crucial baseline for inclusive mathematics education.

Regarding the cognitive profiles and predictors of early algebra performance, the findings of the current study reveal a divergence in the underlying mechanisms utilized by each group. For students without ASD, fluid reasoning and mathematical age emerged as the most significant predictors, a result that echoes established literature identifying general reasoning ability and arithmetical fluency as robust scaffolds for algebraic abstraction (Fuchs et al., 2006; Geary et al., 2007). These results reinforce existing theories in mathematics education that view arithmetic as a conceptual foundation for algebra and highlight the importance of domain-general cognitive abilities in supporting pattern generalization (Green et al., 2017).

In contrast, the cognitive predictors for students with ASD followed a different pattern. Fluid reasoning and mathematical age did not significantly predict performance for the ASD group. Instead, students with ASD relied heavily on spatial reasoning and theory of mind to navigate the pattern generalization task. Within this framework, these findings suggest that spatial reasoning and ToM play a more central role in explaining individual differences in early algebraic thinking in ASD, contributing uniquely to performance compared to basic arithmetical knowledge or other cognitive abilities included in the model.

This reliance on spatial reasoning aligns with the well-documented visuospatial strengths and visually oriented thinking styles frequently observed in autistic individuals (Baron-Cohen, 2002; Bruno et al., 2024; Fernández-Cobos et al., in press). Nevertheless, this study is among the first to link spatial reasoning directly to algebraic pattern generalization in ASD. This finding underscores the potential of visual patterning as a gateway to algebraic understanding for autistic students, especially in tasks involving figural or non-symbolic representations.

Even more novel is the identification of ToM as a predictor of early algebraic thinking in the ASD group, indicating that it helped explaining differences in algebraic performance beyond the other cognitive abilities included in the model. Although ToM is typically framed within the domain of social cognition, these findings suggest that it may also support domain-general inference and abstraction processes essential for identifying structure in patterns. The prominent role of ToM suggests that solving structural mathematical problems may require underlying perspective-taking or the mapping of conceptual relationships between representational spaces (Yang et al., 2019), a pathway that appears

uniquely salient for students with ASD when compensating for traditional domain-general or arithmetical constraints (Polo-Blanco et al., 2024). This extends current discourse in mathematics education by revealing a previously underexplored link between social-cognitive mechanisms and mathematical generalization in neurodiverse learners.

Notably, working memory and processing speed did not emerge as significant predictors in either group, indicating that they did not account for additional variance in early algebraic performance once other cognitive abilities were included in the model. This contrasts with the well-documented role of working memory and processing speed as critical foundations for basic arithmetic, computation, and multi-step word problem solving in both typically developing students and those with ASD (Assouline et al., 2012; Chen et al., 2019; Marinopoulou et al., 2024; Polo-Blanco et al., 2024). However, this finding aligns with emerging evidence that, as tasks shift toward early algebraic thinking, domain-general executive speeds become less salient than other reasoning processes, such as spatial reasoning (e.g., Berg 2008; Chimoni et al. 2023a; Floyd et al. 2003; Tolar et al. 2009). In this study, this shift may reflect the design of the figural pattern task, which emphasized the identification of structural rules and visual pattern recognition over intensive calculation or symbolic manipulation. These results highlight the need to carefully consider task format and structural demands when interpreting how different cognitive abilities support mathematical performance across diverse neurodevelopmental profiles.

Conclusions

This study advances the mathematics education literature by demonstrating that early algebraic thinking is not universally supported by the same cognitive resources across learners. Rather, the cognitive foundations of pattern generalization—the focus of this study—vary meaningfully between non-ASD and ASD students. For non-ASD students, fluid reasoning and early arithmetical abilities provide a foundation for generalizing patterns. For students with ASD, spatial reasoning and theory of mind appear to offer alternative cognitive pathways into early algebraic concepts. These findings have three key implications for research and practice in mathematics education: (a) The results extend current theoretical models of early algebra by identifying theory of mind—not traditionally considered a mathematical predictor—as a meaningful contributor to early algebraic thinking in autistic students. This invites a reconceptualization of what constitutes relevant cognitive support for algebraic thinking; (b) The study supports the view that multiple cognitive routes can lead to successful

algebraic thinking. For neurodiverse learners, particularly those with ASD, visual-spatial strategies may serve as effective entry points into algebra, highlighting the importance of non-symbolic representations in early instruction; (c) The findings advocate for tailored instructional approaches that align with students' cognitive strengths. For autistic learners, visual supports, patterning tasks, and opportunities to engage in inference-based reasoning may be more effective than traditional symbol-based mathematical instruction focused on arithmetic fluency.

Limitations and Future Directions

Several limitations should be acknowledged. First, due to the inherent challenges of recruiting ASD students, this study is constrained by a relatively small sample size and a broad developmental span of 6 to 12 years olds. Running multiple linear regressions with seven predictors limits statistical power. Consequently, the presented models cannot be considered generalizable and should be treated as exploratory pathways for future, larger-scale validation studies.

Second, the study relied on a single functional thinking task involving a growing figural pattern of tables and chairs, which may emphasize certain cognitive processes (e.g., spatial reasoning) over others. This is a task type widely used in previous literature with both typically developing students (e.g., Blanton et al., 2015) and students with ASD (Polo-Blanco et al., 2025), requiring participants to identify an underlying functional relationship and extend it to near and far terms, thereby targeting key processes of early algebraic thinking such as noticing, generalizing, representing, and justifying. As such, despite being a single instrument, it provides a rich, structured assessment of the core dimensions of early algebraic thinking. However, the use of a single measure may still limit construct coverage, and the findings may partially reflect task-specific demands rather than the full complexity of early algebraic thinking.

Third, attention and broader language-related processes, which were not specifically assessed in the present study, may also have influenced task performance. For instance, previous research indicated that verbal comprehension strongly supports mathematical development and word-problem solving in both typically developing students and ASD students (Polo-Blanco et al., 2024; Purpura & Reid, 2016; Viesel-Nordmeyer et al., 2022). While the figural pattern task used in this study was designed to emphasize visual structures over demanding linguistic manipulation, language difficulties common in autism may influence how students interpret problem directions or coordinate linguistic and quantitative information (Bae et al., 2015; Fernández-Cobos et al., in press). Future studies should explicitly

control for language and attentional factors to isolate their unique impact from the cognitive predictors identified here.

Finally, regarding the use of the TEMA-3, it was selected because it assesses early arithmetical competencies, which are considered important precursors of early algebraic thinking and are therefore aligned with the aims of the present study. The TEMA-3 is normed for typically developing children between approximately 3 and 9 years of age and is also designed for use with older children with mathematical difficulties. Moreover, it has been used in research involving autistic participants in the same age range as those included in the present study (e.g., Polo-Blanco et al., 2024). However, in the present study, some older participants reached the maximum score on the TEMA-3, producing a ceiling effect. This limitation should be considered when interpreting the findings involving mathematical age.

Future studies could replicate these findings in larger and more diverse samples, incorporate a broader set of algebraic tasks (e.g., functional thinking, generalized arithmetic), and examine how instructional interventions can be designed to leverage students' individual cognitive profiles. By deepening our understanding of how different learners approach early algebraic thinking, we can move toward a more inclusive and cognitively responsive mathematics education.

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Data Availability The data supporting the findings of this study are available upon request. This study was not preregistered.

Declarations

Conflict of interest The authors declare that they have no conflicts of interest.

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